

Contents

Acknowledgments	xv
Preface	xvii
1. GENERAL IDEAS, CONCEPTS, DEFINITIONS, AND OTHER PRELIMINARY INFORMATION	1
1 ESSENCE OF THE HIERARCHICAL ANALYTICAL-NUMERICAL METHODS	1
1.1 One Illustrative Example of the Multi-Frequency Oscillation-Wave Systems	1
1.2 Essence of the Hierarchical Approach	4
1.3 Classification of the Hierarchical Methods Discussed in the Book	7
2 BASIC CONCEPTS AND DEFINITIONS OF THE OSCILLATION THEORY OF WEAKLY NONLINEAR SYSTEMS	8
2.1 Nonlinear Oscillations and Nonlinear Systems	9
2.2 Hidden and Explicit Oscillation Phases	12
2.3 Resonances	13
2.4 Slowly Varying Amplitudes and Slowly Varying Initial Phases. Complex Amplitudes	20
2.5 Harmonic and Non-Harmonic Oscillations	21
3 BASIC CONCEPTS AND DEFINITIONS OF THE THEORY OF WAVES IN WEAK NONLINEAR SYSTEMS	23
3.1 Definition of the Wave	23
3.2 The Phase and Group Wave Velocities	24
3.3 The Phase of a Wave	26

3.4	Transverse and Longitudinal Waves	26	1.3	Self-Modeling Principle	51
3.5	Surface and Volumetric Waves	27	1.4	Main Idea of the Hierarchical Method	52
3.6	The Concept of Dispersion	27	1.5	Again: What is Hierarchy Originally? The Structural Hierarchy	53
3.7	Waves with Negative, Zero and Positive Energy	28	1.6	Dynamical Hierarchy	54
4	OTHER BASIC DEFINITIONS AND EQUATIONS OF PHYSICS	29	1.7	Hierarchical Series in Dimensionless Form	58
4.1	Linear Velocity and Linear Acceleration	29	2	HIERARCHICAL PRINCIPLES. HIERARCHICAL DESCRIPTION	58
4.2	Linear Momentum and Force	29	2.1	Hierarchical Principles	58
4.3	Curvilinear Motion	30	2.2	Dynamical Equation of the Zeroth Hierarchical Level	60
4.4	Rotation	30	2.3	Structural and Functional (Dynamical) Operators	62
4.5	Energy. Field of Forces	31	2.4	Classification of the Hierarchical Problems	64
4.6	Motion Integrals	32	2.5	Hierarchical Tree	66
4.7	Canonical Variables. Hamiltonian Equations	33	3.	HIERARCHICAL ASYMPTOTIC METHODS. GENERAL IDEAS	71
4.8	Electromagnetic Field	33	1	DETERMINED HIERARCHICAL SYSTEMS	72
4.9	Lagrange and Euler Variables	34	1.1	Determined Hierarchical Systems	72
5	CONCEPT OF SYSTEM	35	1.2	Averaging Operators	76
5.1	What is a System?	35	2	STOCHASTIC HIERARCHICAL SYSTEMS	78
5.2	State of the System	36	2.1	Factors of Stochasticity in Dynamical Systems	78
5.3	General Classification of the Systems	36	2.2	Example of Hierarchical Model of Stochastic System	79
5.4	Some General Properties of Complex Systems	37	3	WAVE RESONANT HIERARCHICAL SYSTEMS	81
6	BASIC POSTULATES AND PRINCIPLES OF THE GENERAL SYSTEM THEORY	38	3.1	Standard Hierarchical Equations in the Case of Hierarchical Wave Problem	82
6.1	Principle of Physicality	38	3.2	Classification of Problems	85
6.2	Integrity postulate	39	3.3	Krylov–Bogolyubov Substitution	87
6.3	Autonomy Postulate	39	3.4	Case b)	88
6.4	Principle of Ability to Model	40	4	VAN DER POL'S METHOD	90
6.5	Complementarity Postulate	40	4.1	A Few Introductory Words	90
6.6	Action Postulate	41	4.2	Van der Pol's Method's Variables	90
6.7	Uncertainty Postulate	41	4.3	Truncated Equations and Their Hierarchical Sense	92
6.8	Purposefulness Principle	42	5	METHODS OF AVERAGING. STANDARD VERSION	95
6.9	Entropy and Negentropy	42	5.1	Bogolyubov's Standard System	95
6.10	Complexity	45	5.2	The Problem of Secular Terms	99
2.	HIERARCHY AND HIERARCHICAL SYSTEMS	49			
1	BASIC CONCEPTS OF THE THEORY OF HIERARCHICAL DYNAMICAL SYSTEMS	49			
1.1	Why Does Nature Need Hierarchy?	49			
1.2	Two Approaches to the Theory of Hierarchical Systems	50			

6	METHODS OF AVERAGING. TWO-LEVEL SYSTEMS WITH SLOW AND FAST VARIABLES	100
6.1	Two-Level Systems with Slow and Fast Variables. General Case	100
6.2	Two-Level Systems with Fast Rotating Phases	102
4.	HIERARCHICAL SYSTEMS WITH FAST ROTATING PHASES	107
1	HIERARCHICAL OSCILLATIONS	107
1.1	A Few Introductory Words	107
1.2	Formulation of the Hierarchical Single-Particle Electrodynamical Problem	109
1.3	Classification of Oscillatory Phases and Resonances. Hierarchical Tree	112
1.4	Reducing Hierarchical Multi-Level Standard System to the Two-Level Form. The Scheme of Hierarchical Transformations	117
2	THE CASE OF SIMPLEST TWO-LEVEL SYSTEM WITH ONE ROTATING SCALAR PHASE	119
2.1	Formulation of the Problem	119
2.2	Algorithm of Asymptotic Integration	120
2.3	Accuracy of Approximate Solutions	124
2.4	Asymptotic Integration of Initial Equations by Means of Successive Approximations	127
2.5	Peculiarities of Asymptotic Hierarchical Calculational Schemes Based on the Fourier Method	128
3	CASE OF TWO FAST ROTATING SCALAR PHASES	130
3.1	Formulation of Problem	130
3.2	Solutions. Non-Resonant Case	131
3.3	Solutions. Resonant Case	134
4	THE CASE OF MANY ROTATING SCALAR PHASES	139
4.1	Formulation of the Problem	139
4.2	Algorithm of Asymptotic Integration	140
5	ALGORITHM FOR SEWING TOGETHER RESONANT AND NON-RESONANT SOLUTIONS	143
5.1	Essence of the Problem	143

5.2	Sewing Together of Resonant and Non-Resonant Solutions	144
5.3	Example for the Solution 'Sewing': The 'Stimulated' Duffing Equation	146
5.	HIERARCHICAL SYSTEMS WITH FAST ROTATING PHASES. EXAMPLES OF PRACTICAL APPLICATIONS	157
1	GENERAL PROPERTIES OF THE MODEL: 'A CHARGED PARTICLE IN THE FIELD OF A STANDING ELECTROMAGNETIC WAVE' AND EXAMPLES OF ITS PRACTICAL REALIZATION	158
1.1	Systems for Transformation of Optical Signals into Microwave Signals as a Convenient Illustrative Examples	158
1.2	Formulation of the Problem of Electron Motion in the Field of Two Oppositely Propagating Electromagnetic Waves	164
2	MODEL 'AN ELECTRON IN THE FIELD OF TWO OPPOSITELY DIRECTED ELECTROMAGNETIC WAVES' AS A TWO-LEVEL HIERARCHICAL OSCILLATIVE SYSTEM	167
2.1	Reducing Initial Motion Equations to the Standard Forms with Two Rotation Phases	167
2.2	Zeroth Hierarchical Level. Parametrical Resonance	170
2.3	Passage to First Hierarchical Level. Nonlinear Pendulum	171
2.4	Nonlinear Pendulum. The Miller-Gaponov Potential	176
2.5	Nonlinear Pendulum. The First Motion Integral	178
2.6	Nonlinear Pendulum. Exact Solutions and Analysis	178
2.7	Full Solutions of the Initial System	183
3	MODEL 'AN ELECTRON IN THE FIELD OF TWO OPPOSITELY DIRECTED ELECTROMAGNETIC WAVES' AS A THREE-LEVEL HIERARCHICAL OSCILLATIVE SYSTEM	185
3.1	Transition to the Second Hierarchical Level	185

3.2	Duffing Oscillator	188
4	MODEL 'AN ELECTRON IN THE FIELD OF THREE ELECTROMAGNETIC WAVES' AS A FOURTH-LEVEL HIERARCHICAL OSCILLATATIVE SYSTEM	192
4.1	Stimulated Oscillation of a Charged Particle	192
4.2	Stimulated Oscillations of an Electron Ensemble	196
6.	HIERARCHICAL SYSTEMS WITH PARTIAL DERIVATIVES. METHOD OF AVERAGED CHARACTERISTICS	207
1	SOME PRELIMINARY INFORMATION	208
1.1	Motion Equations	208
1.2	Field Equations	210
1.3	Some General Information about Equations with Partial Derivatives	211
2	METHOD OF AVERAGED CHARACTERISTICS	213
2.1	Concept of the Standard Form	213
2.2	General Scheme of the Method	214
3	CHARACTERISTICS AND THE METHOD OF CHARACTERISTICS	222
3.1	Method of Characteristics. The Scalar Case	222
3.2	Method of Characteristics. The Vector Case	227
4	EXAMPLE: APPLICATION OF THE METHOD OF AVERAGED CHARACTERISTICS FOR A SIMPLEST SYSTEM WITH OSCILLATATIVE RIGHT PARTS	231
4.1	Initial Equations	231
4.2	Characteristics	232
4.3	Passage to the First Hierarchical Level	232
4.4	Back Transformations	234
5	HIERARCHICAL METHOD OF AVERAGED QUASI-HYDRODYNAMIC EQUATION	236
5.1	Averaged Quasi-Hydrodynamic Equation	237
5.2	Back Transformations	240
6	THE METHOD OF AVERAGED CURRENT-DENSITY EQUATION	240
6.1	Averaged Current-Density Equation	241
6.2	Back Transformations	244

7	HIERARCHICAL METHOD OF THE AVERAGED KINETIC EQUATION	244
7.1	Averaged Kinetic Equation	244
7.	EXAMPLE: APPLICATION OF THE METHOD OF AVERAGED CHARACTERISTICS IN NONLINEAR THEORY OF THE TWO-STREAM INSTABILITY	249
1	PROBLEM OF MOTION OF A TWO-VELOCITY ELECTRON BEAM IN GIVEN ELECTROMAGNETIC FIELDS	252
1.1	Statement of the Motion Problem	253
1.2	Averaged Characteristics for the Motion Problem	254
1.3	Back Transformations	260
1.4	Integration of the Averaged Quasi-Linear Equation for the Beam Velocity	261
2	FIELD PROBLEM. HIERARCHICAL ASYMPTOTIC INTEGRATION OF THE CONTINUITY EQUATION	262
2.1	Continuity Equation of the Two-Velocity Electron Beam	263
2.2	Averaged Characteristics and the Averaged Quasi-Linear Equation	264
2.3	Back Transformation	268
2.4	Characteristics of the Averaged Continuity Equation	270
3	FIELD PROBLEM. APPLICATION OF THE METHOD OF AVERAGED CHARACTERISTICS FOR ASYMPTOTIC INTEGRATION OF THE MAXWELL'S EQUATIONS	271
3.1	Averaged Maxwell's Equations	271
3.2	Back Transformations	275
3.3	Solving the Averaged Quasilinear Equation for the Electric Field	276
3.4	Truncated Equations for the Harmonic Amplitudes of a Space Charge Wave	277
3.5	Some Commentaries for the Obtained Results	279
8.	HIERARCHICAL SYSTEMS WITH PARTIAL DERIVATIVES. SOME OTHER ASYMPTOTIC METHODS	283

1	MAIN IDEAS OF THE METHOD OF SLOWLY VARYING AMPLITUDES	284
1.1	General Computational Scheme of the Slowly Varying Amplitudes Method	285
1.2	Simplified Version of the Slowly Varying Amplitude Method. Example: Effect of Parametric Amplification of a Wave	288
2	TRADITIONAL VARIANT OF THE SLOWLY VARYING AMPLITUDES METHOD. RIGOROUS VERSION	293
2.1	Case of Spatially One-Dimensional Model	294
2.2	Classification of Transversely Inhomogeneous Models	306
2.3	Model with Moderate Inhomogeneity	306
2.4	Method of Parabolic Equation	308
3	MODERNIZED VERSION OF THE SLOWLY VARYING AMPLITUDE METHOD	309
3.1	Field Problem	309
3.2	Current Density Problem	311
3.3	Current Density Problem in Framework of the Kinetic Approach	313
4	METHOD OF HIERARCHICAL TRANSFORMATION OF COORDINATES	314
4.1	Main Idea of the Hierarchical Transformations	314
4.2	Hierarchical Equations	317
4.3	Averaged Operator $\bar{\nabla}$	320
5	MITROPOL'SKII METHOD	322
5.1	Reduction of a Partial Differential Equation to the Standard Form with Fast Rotating Phases	322
5.2	Basic Solutions	323
5.3	Truncated Equations	324
6	EXAMPLES OF REDUCING OF THE MAXWELL EQUATIONS TO THE STANDARD FORM FOR THE METHOD OF SLOWLY VARYING AMPLITUDES	326
6.1	Kinetic Version	326
6.2	Quasi-Hydrodynamic Case	328

7	EXAMPLE: THE TWO-STREAM INSTABILITY IN A TWO-VELOCITY ELECTRON BEAM. THE METHODS OF AVERAGED QUASI-HYDRODYNAMIC EQUATION AND SLOWLY VARYING AMPLITUDES	329
7.1	Statement of the Problem	329
7.2	Motion Problem. The Averaged Quasi-hydrodynamic Equation	330
7.3	Motion Problem. The Back Transformations	340
7.4	Field Problem. The Method of Slowly Varying Amplitudes	341
	Appendices	349
	Results of calculations in the second approximation	349
	Index	351