

Contents

Foreword.	xiii
<i>J.-P. Kahane</i>	
Chapter 1. Introduction.	1
<i>G. Cherbit</i>	
Chapter 2. Sundry observations	3
<i>B. Mandelbrot</i>	
Bibliography	9
Chapter 3. Models of irregular curves	10
<i>S. Dubuc</i>	
3.1 Examples of irregular curves	11
3.1.1 Curves associated with trigonometric series (11)	
3.1.2 Von Koch–Mandelbrot curves (12)	
3.1.3 Hyperbolic curves (15)	
3.2 Curves modelled by a functional equation	17
3.3 Notions of dimension.	18
3.3.1 Hausdorff dimension (18)	
3.3.2 Dimension of internal similarity (19)	
3.3.3 Bouligand’s dimensional order (19)	
3.3.4 The ε -entropy and ε -capacity of Kolmogoroff (20)	
3.3.5 Other notions of dimension (20)	
3.4 Calculating the size of a curve	21
3.5 Applications	22
References	24
Chapter 4. Dyadic interpolation	27
<i>G. Deslauriers and S. Dubuc</i>	
4.1 Elementary properties of dyadic interpolation	27
4.2 Rapid calculation of the fundamental function	29
4.3 Schwartz distributions and dyadic interpolation	30
4.4 Applications to the interpolation scheme	33
References	35

viii	Contents	
Chapter 5.	Stochastic processes and covering procedures	36
	<i>M. Weber</i>	
5.1	Some covering procedures—their interrelationships and examples	36
5.1.1	Hausdorff measure and dimension (36)	
5.1.2	Some applications (39)	
5.1.3	Capacity dimension (42)	
5.1.4	The dimensions of Schnirelman–Pontjrajin and of Kolmogorov (44)	
5.1.5	The Menger–Urysohn dimension (45)	
5.2	Applications to the study of the regularity of stochastic processes	46
5.2.1	(46)	
5.2.2	(52)	
5.2.3	Ultrametric structures (55)	
References	58
Chapter 6.	Attractors and dimensions	60
	<i>P. Girault</i>	
6.1	Introduction	60
6.2	Some definitions	61
6.2.1	Hausdorff measure and dimension (61)	
6.2.2	Fractal dimension (61)	
6.2.3	The notion of attractors (62)	
6.3	Some theorems on the dimension of attractors	65
6.4	The Lorenz equations.	68
6.5	Some point transformations	71
6.5.1	The Henon attractor (71)	
6.5.2	An attractor with non-empty interior (74)	
6.6	The Navier–Stokes equations	74
References	81
Chapter 7.	Construction of fractals and dimension problems	83
	<i>F. M. Dekking</i>	
7.1	A mathematical description of fractals	83
7.2	Self-similarity properties.	88
7.3	Problems associated with determining the Hausdorff dimension	93
7.3.1	Solubility (93)	
7.3.2	Self-affine fractals (94)	
7.3.3	Piecewise self-affine fractals (96)	
References	96
Chapter 8.	Introduction to packing measures and dimensions	98
	<i>J. Peyrière</i>	
8.1	Introduction	98
8.2	Hausdorff measure and dimension	99
8.3	Packing measures and dimension	100
8.4	A method for determining dim and Dim	101
8.5	The case of brownian trajectories	102

	Contents	ix
Chapter 9.	Some remarks on the Hausdorff dimension	103
	<i>J.-L. Jonot</i>	
9.1	The spaces \mathbf{R}_v^N	108
9.1.1	The valued field \mathbf{R}_v (108)	
9.1.2	Representation in \mathbf{R}_v^2 of the unit circle (109)	
9.1.3	The normed vector space \mathbf{R}_v^N (109)	
9.2	Hausdorff dimension of a subset of \mathbf{R}^N	112
9.3	Volume interpretation of the Hausdorff dimension	115
Chapter 10.	Fractals, materials and energy	120
	<i>A. Le Méhauté</i>	
10.1	Fractals: where are they to be found?	120
10.2	On scales of measurement	124
10.3	The TEISI model	127
10.4	Fractality and dissipation	128
10.5	Why should the world be fractal?	130
10.6	Conclusion	134
Chapter 11.	Problems concerning the concept of fractal in electrochemistry	135
	<i>M. Keddam</i>	
11.1	Foreword	135
11.1.1	Some <i>a priori</i> considerations concerning the involvement of the fractal dimension in electrochemistry (136)	
11.1.2	Some comments on the TEISI model (136)	
11.2	Theoretical considerations	137
11.2.1	Summary (140)	
11.3	Experimental considerations.	140
References	141
Chapter 12.	Some remarks concerning the structure of galactic clusters and Hubble's constant	142
	<i>P. Mills</i>	
References	144
Chapter 13.	Disorder, chance and fractals in biology	145
	<i>G. Cherbit</i>	
13.1	Glomerular filtration	145
13.2	Perturbation of the rate of growth of a culture	147
13.3	Bacterial division	149
Bibliography	150
Chapter 14.	Fractals, semi-fractals and biometry	151
	<i>J.-P. Rigaut</i>	
14.1	Introduction	151
14.2	History	152

14.3	Experimental methods	155
14.3.1	What are we looking for? (155)	
14.3.2	Variation of the perimeter as a function of the resolution (155)	
14.3.3	Methods based on other principles (157)	
14.3.4	Our method for studying the outlines of objects (158)	
14.4	A semi-fractal model	159
14.4.1	Origins (159)	
14.4.2	A semi-fractal equation (161)	
14.4.3	Interpretation of the equation—the semi-fractal model (163)	
14.5	The lung as a semi-fractal object	165
14.5.1	From the lung to steel via the geranium (165)	
14.5.2	Alveolar outlines are semi-fractal (165)	
14.5.3	A semi-fractal object possesses outlines with infinitely self-similar convolutions (167)	
14.5.4	A new model for the alveolar structure (169)	
14.5.5	A semi-fractal object can exhibit a discontinuity in its structure (169)	
14.5.6	Curvatures—relevant or not? (171)	
14.6	A gallery of semi-fractal monsters	171
14.6.1	Motivation (171)	
14.6.2	A curious coincidence? (171)	
14.6.3	A rudimentary semi-fractal monster (172)	
14.6.4	The semi-fractal monster . . . of Archimedes (172)	
14.6.5	Towards ‘coherent’ semi-fractal monsters (173)	
14.6.6	What do our monsters resemble? (174)	
14.7	The frustration of the little quadrat	174
14.7.1	A curious experimental fact (174)	
14.7.2	Theoretical considerations in the setting of the stationary model (176)	
14.7.2	The stationary model is not applicable to our observations (178)	
14.7.4	An explanation of fractal type: the ‘frustrated little quadrat’ (178)	
14.8	Discussion	182
References	184
Chapter 15. Reconstruction of images from projections		188
<i>N. de Beaucoudary, L. Garnero and J.-P. Hugonin</i>		
15.1	Data-collection methods using separate sections	189
15.1.1	Absorption (189)	
15.1.2	Emission (189)	
15.1.3	Projections (191)	
15.1.4	The principle of the reconstruction (191)	
15.2	Tomographic reconstruction of images using the three-dimensional Radon transform	195
15.2.1	Emission (195)	
15.2.2	Another example (195)	
15.3	Codings with non-separated sections and not reducible to the three-dimensional Radon transform	197
Chapter 16. Creation of fractal objects by diffusion		203
<i>M. Rosso, B. Sapoval, J.-F. Gouyet and J.-D. Colonna</i>		
16.1	Introduction	203
16.2	Internal similarity and fractal properties of the diffusion front	204

16.3	Dynamic behaviour of the diffusion front	207
16.4	Conclusion	211
References	211
Chapter 17. Irreversibility and the arrow of time		212
<i>J. Chanu</i>		
Bibliography	213
Chapter 18. Thermodynamic entropy and information		214
<i>M. Courbage</i>		
18.1	Information and thermodynamics	214
18.2	Distance—information from one probability distribution with respect to another	215
18.3	Concluding remarks	220
References	220
Chapter 19. Dimension and entropy of regular curves		222
<i>M. Mendès-France</i>		
19.1	By way of introduction	222
19.2	Dimensions of plane curves	223
19.3	Soluble curves	225
19.4	Entropy of finite curves	226
19.5	Entropy of an infinite curve	227
References	229
Chapter 20. Local dimension, momentum and trajectories		231
<i>G. Cherbit</i>		
20.1	Local dimension—generalized velocity	235
20.1.1	α -Differentiability (235)	
20.1.2	Local dimension (235)	
20.1.3	α -Velocity (236)	
20.1.4	The notion of D -length (236)	
20.2	Summary	237
References	238
Chapter 21. Space–time dimensionality		239
<i>G. Cherbit</i>		
References	242
Hausdorff seminars		244
Index		245