Contents

Pr	eface		xv				
Gl	ossary	of Symbols	xvii				
Gl	ossary	of Abbreviations	xix				
I	Preli	minaries	1				
1	Intro	duction	3				
	1.1	Dynamical Systems	3				
	1.2	Iterative Root-Finding	7				
	1.3	Iterative Fixed-Point Algorithms	8				
	1.4	Convergence Theorems	8				
	1.5	Positivity Constraints	9				
	1.6	Fundamental Concepts	9				
2	Back	ground	11				
	2.1	Iterative Algorithms and their Applications	11				
	2.2	A Basic Inverse Problem	12				
	2.3	Some Applications	12				
	2.4	The Urn Model for Remote Sensing	16				
3	Basic Concepts						
	3.1	The Geometry of Euclidean Space	21				
	3.2	Hyperplanes in Euclidean Space	24				
	3.3	Convex Sets in Euclidean Space	24				
	3.4	Basic Linear Algebra	25				
	3.5	Linear and Nonlinear Operators	29				
	3.6	Exercises	33				

vii	i		Contents
4	Metr	ric Spaces and Norms	35
	4.1	Metric Spaces	. 35
	4.2	Analysis in Metric Space	. 36
	4.3	Norms	. 37
	4.4	Eigenvalues and Eigenvectors	. 38
	4.5	Matrix Norms	. 42
	4.6	Exercises	. 47
II	Ove	erview	49
5	Oper	rators	51
	5.1	Operators	. 51
	5.2	Two Useful Identities	. 52
	5.3	Strict Contractions	. 52
	5.4	Orthogonal Projection Operators	. 54
	5.5	Averaged Operators	. 55
	5.6	Affine Linear Operators	. 58
	5.7	Paracontractive Operators	. 58
	5.8	Exercises	. 62
6	Prob	lems and Algorithms	63
	6.1	Systems of Linear Equations	. 63
	6.2	Positive Solutions of Linear Equations	. 67
	6.3	Sensitivity to Noise	. 69
	6.4	Convex Sets as Constraints	. 69
	6.5	Algorithms Based on Orthogonal Projection	. 71
	6.6	Steepest Descent Minimization	. 72
	6.7	Bregman Projections and the SGP	. 74
	6.8	Applications	. 76
III	Op	perators	77
_			
7	Aver	aged and Paracontractive Operators	79
	7.1	Solving Linear Systems of Equations	. 79
	7.2	Averaged Operators	. 81
	7.3	Paracontractive Operators	. 85
	7.4	Linear and Affine Paracontractions	. 87
	7.5	Other Classes of Operators	. 90

Contents			
IV	Algo	orithms	93
8	The A	Algebraic Reconstruction Technique	95
	8.1	The ART	95
	8.2	When $Ax = b$ Has Solutions $\ldots \ldots \ldots \ldots \ldots \ldots \ldots$	98
	8.3	When $Ax = b$ Has No Solutions $\ldots \ldots \ldots \ldots \ldots$	98
	8.4	Regularized ART	101
	8.5	Avoiding the Limit Cycle	102
9	Simu	ltaneous and Block-Iterative ART	105
	9.1	Cimmino's Algorithm	105
	9.2	The Landweber Algorithms	106
	9.3	The Block-Iterative ART	108
	9.4	The Rescaled Block-Iterative ART	109
	9.5	Convergence of the RBI-ART	109
	9.6	Using Sparseness	111
10	Jacob	i and Gauss-Seidel Methods	113
	10.1	The Jacobi and Gauss-Seidel Methods: An Example	113
	10.2	Splitting Methods	114
	10.3	Some Examples of Splitting Methods	115
	10.4	The Jacobi Algorithm and JOR	116
	10.5	The Gauss-Seidel Method and SOR	118
11	Conj	ugate-Direction Methods in Optimization	123
	11.1	Iterative Minimization	123
	11.2	Quadratic Optimization	124
	11.3	Conjugate Bases for R^J	126
	11.4	The Conjugate Gradient Method	129
	11.5	Exercises	131
v	Posit	tivity in Linear Systems	133
12	The l	Multiplicative ART (MART)	135
	12.1	A Special Case of MART	135
	12.2	MART in the General Case	136
	12.3	ART and MART as Sequential Projection Methods	137
	12.4	Proof of Convergence for MART	140
	12.5	Comments on the Rate of Convergence of MART	141

х

Contents

13.1	Overview					
13.2	The SMART and the EMML Algorithm					
13.3	Ordered-Subset Versions				•	
13.4	The RBI-SMART					
13.5	The RBI-EMML					
13.6	RBI-SMART and Entropy Maximization					

VI Sta	bility	159
14 Sensi	tivity to Noise	161
14.1	Where Does Sensitivity Come From?	161
14.2	Iterative Regularization	165
14.3	A Bayesian View of Reconstruction	166
14.4	The Gamma Prior Distribution for x	167
14.5	The One-Step-Late Alternative	168
14.6	Regularizing the SMART	169
14.7	De Pierro's Surrogate-Function Method	169
14.8	Block-Iterative Regularization	171
15 Feed	back in Block-Iterative Reconstruction	173
15.1	Feedback in ART	174
15.2	Feedback in RBI methods	174
VII Oj	otimization	181
16 Iterat	tive Optimization	183
16.1	Functions of a Single Real Variable	183
16.2	Functions of Several Real Variables	184
16.3	Gradient Descent Optimization	187
16.4	The Newton-Raphson Approach	188
16.5	Rates of Convergence	189
16.6	Other Approaches	190
17 Conv	vex Sets and Convex Functions	191
17.1	Optimizing Functions of a Single Real Variable	191
17.2	Optimizing Functions of Several Real Variables	195
17.3	Convex Feasibility	200
17.4	Optimization over a Convex Set	203
17.5	Geometry of Convex Sets	203
17.6	Projecting onto Convex Level Sets	204
17.7	Projecting onto the Intersection of Convex Sets	205

Contents

18 Generalized Projections onto Convex Sets 18.1 Bregman Functions and Bregman Distances 18.2 The Successive Generalized Projections Algorithm 18.3 Bregman's Primal-Dual Algorithm 18.4 Dykstra's Algorithm for Bregman Projections 19 The Split Feasibility Problem 19.1 The CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 19.1 Noreau's Proximity Operators 20.1 Moreau's Proximity Operators 20.2 Forward-Backward Splitting 20.3 Proximity Operators using Bregman Distances 20.4 The Interior-Point Algorithm (IPA) 20.5 Computing the Iterates 20.6 Some Examples 21.1 Multiple-Distance Successive Generalized Projection 21.2 An Interior-Point Algorithm (IPA) 21.3 The MSGP Algorithm 21.4 An Interior-Point Algorithm for Iterative Optimization 22.1 Primal and Dual Problems 22.3 Convex Programming 23.1 Projection onto Convex Sets 23.2 Solving $Ax = b$ 23.3 The Agmon-Motzkin-Schoenberg Algorithm 24.1 Modifying the KL Distance 25.2 Fourier Transform Estimation 26.3 Fourier-Transform Estimation 25.4 The Limited-Fourier-Data Problem 25.4 The Discrete PDFT (DPDFT)		
18.1 Bregman Functions and Bregman Distances 18.2 The Successive Generalized Projections Algorithm 18.3 Bregman's Primal-Dual Algorithm 18.4 Dykstra's Algorithm for Bregman Projections 18.4 Dykstra's Algorithm for Bregman Projections 19.1 The CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 20 Nonsmooth Optimization 20.1 Moreau's Proximity Operators 20.2 Forward-Backward Splitting 20.3 Proximity Operators using Bregman Distances 20.4 The Interior-Point Algorithm (IPA) 20.5 Computing the Iterates 20.6 Some Examples 21 An Interior-Point Optimization Method 21.1 Multiple-Distance Successive Generalized Projection 21.2 An Interior-Point Algorithm for Iterative Optimization 22 In MSGP Algorithm 21.3 The MSGP Algorithm for Iterative Optimization 22 Linear and Convex Programming 22.1 Primal and Dual Problems 22.2 The Simplex Method 23.	18	Generalized Projections onto Convex Sets
18.2 The Successive Generalized Projections Algorithm 18.3 Bregman's Primal-Dual Algorithm 18.4 Dykstra's Algorithm for Bregman Projections 18.4 Dykstra's Algorithm for Bregman Projections 19 The Split Feasibility Problem 19.1 The CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 20 Nonsmooth Optimization 20.1 Moreau's Proximity Operators 20.2 Forward-Backward Splitting 20.3 Proximity Operators using Bregman Distances 20.4 The Interior-Point Algorithm (IPA) 20.5 Computing the Iterates 20.6 Some Examples 21 An Interior-Point Optimization Method 21.1 Multiple-Distance Successive Generalized Projection 21.2 An Interior-Point Algorithm for Iterative Optimization 21.3 The MSGP Algorithm 21.4 An Interior-Point Algorithm for Iterative Optimization 22.1 Primal and Dual Problems 22.2 The Simplex Method 22.3 Convex Programming 22.4 The Agmon-Motzkin-Schoenberg Algorithm 23.3		18.1 Bregman Functions and Bregman Distances
 18.3 Bregman's Primal-Dual Algorithm		18.2 The Successive Generalized Projections Algorithm
18.4 Dykstra's Algorithm for Bregman Projections 19 The Split Feasibility Problem 19.1 The CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 20 Nonsmooth Optimization 20.1 Moreau's Proximity Operators 20.2 Forward-Backward Splitting 20.3 Proximity Operators using Bregman Distances 20.4 The Interior-Point Algorithm (IPA) 20.5 Computing the Iterates 20.6 Some Examples 21 An Interior-Point Optimization Method 21.1 Multiple-Distance Successive Generalized Projection 21.2 An Interior-Point Algorithm (IPA) 21.3 The MSGP Algorithm 21.4 An Interior-Point Algorithm for Iterative Optimization 22 Linear and Convex Programming 22.1 Primal and Dual Problems 22.2 The Simplex Method 23.3 Convex Programming 24.4 Nodifying the KL Distance 23.3 The Agmon-Motzkin-Schoenberg Algorithm 24 Constrained Iteration Methods		18.3 Bregman's Primal-Dual Algorithm
19 The Split Feasibility Problem 19.1 The CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 19.2 Particular Cases of the CQ Algorithm 20 Nonsmooth Optimization 20.1 Moreau's Proximity Operators 20.2 Forward-Backward Splitting 20.3 Proximity Operators using Bregman Distances 20.4 The Interior-Point Algorithm (IPA) 20.5 Computing the Iterates 20.6 Some Examples 21 An Interior-Point Optimization Method 21.1 Multiple-Distance Successive Generalized Projection 21.2 An Interior-Point Algorithm (IPA) 21.3 The MSGP Algorithm 21.4 An Interior-Point Algorithm for Iterative Optimization 21.5 The Simplex Method 21.1 Primal and Dual Problems 22.2 The Simplex Method 23.3 Convex Programming 24.1 Projection onto Convex Sets 23.2 Solving $Ax = b$ 23.3 The Agmon-Motzkin-Schoenberg Algorithm 24.1 Modifying the KL Distance 24.2 The ABMART Algorithm 24.3 The ABEMML Algorithm 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.2 Minimum-Norm Estimation 25.3 Fourier-Transform Data		18.4 Dykstra's Algorithm for Bregman Projections
19The Split Feasibility Problem19.1The CQ Algorithm19.2Particular Cases of the CQ Algorithm19.2Particular Cases of the CQ Algorithm20.1Moreau's Proximity Operators20.2Forward-Backward Splitting20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples21An Interior-Point Optimization Method21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization2121.121.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method23.3Convex Programming24.4Projection onto Convex Sets23.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.2The ABMART Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)	• •	
19.1The CQ Algorithm19.2Particular Cases of the CQ Algorithm19.2Particular Cases of the CQ Algorithm20.1Moreau's Proximity Operators20.2Forward-Backward Splitting20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples20.7An Interior-Point Optimization Method21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method23.3Convex Programming24.4Constrained Iteration Methods24.1Modifying the KL Distance24.2The ABMART Algorithm24.3The ABEMML Algorithm25.4The Limited-Fourier-Data Problem25.3Fourier-Transform Estimation25.4The Limited-Fourier-Data Problem25.4The Limited-Fourier-Data Problem25.4The Discrete PDFT (DPDFT)	19	The Split Feasibility Problem
19.2Particular Cases of the CQ Algorithm20Nonsmooth Optimization20.1Moreau's Proximity Operators20.2Forward-Backward Splitting20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples21An Interior-Point Optimization Method21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.2In Interior-Point Algorithm for Iterative Optimization21.4An Interior-Point Algorithm for Iterative Optimization22The Simplex Method23.1Primal and Dual Problems24.2The Simplex Method25.3Solving $Ax = b$ 26.3Systems of Linear Inequalities27.4Projection onto Convex Sets28.4Solving $Ax = b$ 29.4The Agmon-Motzkin-Schoenberg Algorithm24.4Modifying the KL Distance24.2The ABMART Algorithm25.4The Limited-Fourier-Data Problem25.3Fourier Transform Estimation25.4The Limited-Fourier-Data Problem25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		19.1 The CQ Algorithm
20Nonsmooth Optimization20.1Moreau's Proximity Operators20.2Forward-Backward Splitting20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples21An Interior-Point Optimization Method21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.2In Interior-Point Algorithm for Iterative Optimization21.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method23.3Convex Programming23.4Systems of Linear Inequalities23.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.2The ABMART Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		19.2 Particular Cases of the CQ Algorithm
20.1Moreau's Proximity Operators20.2Forward-Backward Splitting20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method23.3Convex Programming24.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.2The ABMART Algorithm25Fourier Transform Estimation25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)	20	Nonsmooth Optimization
20.1Initial of Proximity Operators using Bregman Distances20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples20.7Multiple-Distance Successive Generalized Projection21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.5Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method23.3Convex Programming23.4Systems of Linear Inequalities23.3The Agmon-Motzkin-Schoenberg Algorithm24.4Modifying the KL Distance24.2The ABMART Algorithm24.3The ABEMML Algorithm25Fourier Transform Estimation25.4The Limited-Fourier-Data Problem25.3Fourier-Transform Data26.4The Discrete PDFT (DPDFT)	20	20.1 Moreau's Proximity Operators
20.2Forward-Dackward Splitting20.3Proximity Operators using Bregman Distances20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples20.7Multiple-Distance Successive Generalized Projection21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method23.3Convex Programming24.3Convex Programming25.4Solving $Ax = b$ 26.5Solving $Ax = b$ 27.4The Agmon-Motzkin-Schoenberg Algorithm28.4The ABEMML Algorithm29.5Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.3Fourier-Transform Data26.4The Discrete PDFT (DPDFT)		20.2 Forward Backward Splitting
20.3From Problem20.4The Interior-Point Algorithm (IPA)20.5Computing the Iterates20.6Some Examples20.7Some Examples21An Interior-Point Optimization Method21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization2121.121.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method22.3Convex Programming23.4Projection onto Convex Sets23.5Systems of Linear Inequalities23.1Projection onto Convex Sets23.2Solving $Ax = b$ 24Constrained Iteration Methods24.1Modifying the KL Distance24.2The ABMART Algorithm24.3The ABEMML Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)25.4The Discrete PDFT (DPDFT)		20.2 Provinity Operators using Brogman Distances
20.4The Interior-Fonte Algorithm (IFA)20.5Computing the Iterates20.6Some Examples21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IFA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21Innerior-Point Algorithm for Iterative Optimization21.4An Interior-Point Algorithm for Iterative Optimization21.5Convex Programming22.6Primal and Dual Problems22.7The Simplex Method22.8Convex Programming23.1Projection onto Convex Sets23.2Solving $Ax = b$ 23.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.3The ABMART Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		20.0 Troumity Operators using Disginan Distances
20.5Computing the iterates20.6Some Examples21An Interior-Point Optimization Method21.1Multiple-Distance Successive Generalized Projection21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method22.3Convex Programming22.4The Simplex Method22.5Convex Programming23Systems of Linear Inequalities23.1Projection onto Convex Sets23.2Solving $Ax = b$ 23.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.3The ABMART Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.2Minimum-Norm Estimation25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		20.4 The Interfor-Point Algorithm (IFA)
 20.6 Some Examples		20.5 Computing the Iterates
 21 An Interior-Point Optimization Method 21.1 Multiple-Distance Successive Generalized Projection 21.2 An Interior-Point Algorithm (IPA)		20.6 Some Examples
21.1 Multiple-Distance Successive Generalized Projection	21	An Interior-Point Optimization Method
21.2An Interior-Point Algorithm (IPA)21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization22Linear and Convex Programming22.1Primal and Dual Problems22.2The Simplex Method22.3Convex Programming22.3Convex Programming23.4Systems of Linear Inequalities23.1Projection onto Convex Sets23.2Solving $Ax = b$ 23.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.3The ABEMML Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		21.1 Multiple-Distance Successive Generalized Projection
21.2The MSGP Algorithm21.3The MSGP Algorithm21.4An Interior-Point Algorithm for Iterative Optimization21.4An Interior-Point Algorithm for Iterative Optimization21.5Primal and Dual Problems22.6The Simplex Method22.7The Simplex Method22.8Convex Programming22.9Convex Programming23.1Projection onto Convex Sets23.2Solving $Ax = b$ 23.3The Agmon-Motzkin-Schoenberg Algorithm24Constrained Iteration Methods24.1Modifying the KL Distance24.2The ABMART Algorithm24.3The ABEMML Algorithm25Fourier Transform Estimation25.1The Limited-Fourier-Data Problem25.2Minimum-Norm Estimation25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		21.2 An Interior-Point Algorithm (IPA)
21.0The histor Highlight for Heritic Contraction in the intervention of the inte		21.3 The MSGP Algorithm
 21.1 Init interfer Fourier Trigorithm for Terreture of primitation in the restaurce of the rest		21.4 An Interior-Point Algorithm for Iterative Ontimization
 22 Linear and Convex Programming 22.1 Primal and Dual Problems 22.2 The Simplex Method 22.3 Convex Programming 22.3 Convex Programming 23 Systems of Linear Inequalities 23.1 Projection onto Convex Sets 23.2 Solving Ax = b 23.3 The Agmon-Motzkin-Schoenberg Algorithm 24 Constrained Iteration Methods 24.1 Modifying the KL Distance 24.2 The ABMART Algorithm 24.3 The ABEMML Algorithm 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.3 Fourier-Transform Data 25.4 The Discrete PDFT (DPDFT) 		
 22.1 Primal and Dual Problems	22	Linear and Convex Programming
 22.2 The Simplex Method		22.1 Primal and Dual Problems
 22.3 Convex Programming		22.2 The Simplex Method
 23 Systems of Linear Inequalities 23.1 Projection onto Convex Sets 23.2 Solving Ax = b 23.3 The Agmon-Motzkin-Schoenberg Algorithm 24 Constrained Iteration Methods 24.1 Modifying the KL Distance 24.2 The ABMART Algorithm 24.3 The ABEMML Algorithm 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.3 Fourier-Transform Data 25.4 The Discrete PDFT (DPDFT) 		22.3 Convex Programming
 23 Systems of Linear Inequalities 23.1 Projection onto Convex Sets	~~	
 23.1 Projection onto Convex Sets	23	Systems of Linear Inequalities
 23.2 Solving Ax = b		23.1 Projection onto Convex Sets
 23.3 The Agmon-Motzkin-Schoenberg Algorithm 24 Constrained Iteration Methods 24.1 Modifying the KL Distance		23.2 Solving $Ax = b$
 24 Constrained Iteration Methods 24.1 Modifying the KL Distance 24.2 The ABMART Algorithm 24.3 The ABEMML Algorithm 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.2 Minimum-Norm Estimation 25.3 Fourier-Transform Data 25.4 The Discrete PDFT (DPDFT) 		23.3 The Agmon-Motzkin-Schoenberg Algorithm
24.1 Modifying the KL Distance 24.2 The ABMART Algorithm 24.3 The ABEMML Algorithm 24.3 The ABEMML Algorithm 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.2 Minimum-Norm Estimation 25.3 Fourier-Transform Data 25.4 The Discrete PDFT (DPDFT)	24	Constrained Iteration Methods
24.2 The ABMART Algorithm	- 1	24.1 Modifying the KL Distance
24.2 The ABEMML Algorithm 24.3 The ABEMML Algorithm 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.2 Minimum-Norm Estimation 25.3 Fourier-Transform Data 25.4 The Discrete PDFT (DPDFT)		24.2 The ABMART Algorithm
 25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem		24.2 The ABEMMI Algorithm
25 Fourier Transform Estimation 25.1 The Limited-Fourier-Data Problem 25.2 Minimum-Norm Estimation 25.3 Fourier-Transform Data 25.4 The Discrete PDFT (DPDFT)		24.5 THE ADDIVINIL AIGOIDHIII
25.1The Limited-Fourier-Data Problem25.2Minimum-Norm Estimation25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)	25	Fourier Transform Estimation
25.2Minimum-Norm Estimation25.3Fourier-Transform Data25.4The Discrete PDFT (DPDFT)		25.1 The Limited-Fourier-Data Problem
25.3 Fourier-Transform Data		25.2 Minimum-Norm Estimation
25.4 The Discrete PDFT (DPDFT)		25.3 Fourier-Transform Data
		25.4 The Discrete PDFT (DPDFT)

xi

xii	Contents
VIII Applications	269
26 Tomography	271
26.1 X-Ray Transmission Tomography	271
26.2 Emission Tomography	275
26.3 Image Reconstruction in Tomography	276
27 Intensity-Modulated Radiation Therapy	279
27.1 The Extended CQ Algorithm	279
27.2 Intensity-Modulated Badiation Therapy	280
27.3 Equivalent Uniform Dosage Functions	280
21.0 Equivalent of form Dosage Functions	200
28 Magnetic-Resonance Imaging	283
28.1 An Overview of MRI	283
28.2 Alignment	284
28.3 Slice Isolation	284
28.4 Tipping	284
28.5 Imaging	285
28.6 The General Formulation	287
28.7 The Received Signal	287
29 Hyperspectral Imaging	291
29.1 Spectral Component Dispersion	291
20.2 A Single Point Source	· · 201 202
20.3 Multiple Point Sources	· · 202
29.4 Solving the Mixture Problem	233
29.4 Solving the Mixture Problem	234
30 Planewave Propagation	295
30.1 Transmission and Remote Sensing	295
30.2 The Transmission Problem	296
30.3 Reciprocity	297
30.4 Remote Sensing	297
30.5 The Wave Equation	297
30.6 Planewave Solutions	298
30.7 Superposition and the Fourier Transform	299
30.8 Sensor Arrays	300
30.9 The Remote-Sensing Problem	301
30.10 Sampling	302
30.11 The Limited-Aperture Problem	
30.12 Resolution	304
30 13 Discrete Data	306
30 14 The Finite-Data Problem	307
JOIT THE LUME DATA LIQUEM	

Cor	ntents	xiii
	30.15 Functions of Several Variables	308 309
31	Inverse Problems and the Laplace Transform31.1 The Laplace Transform and the Ozone Layer31.2 The Laplace Transform and Energy Spectral Estimation	311 311 313
32	Detection and Classification32.1Estimation32.2Detection32.3Discrimination32.4Classification32.5More Realistic Models	 315 315 318 320 321 322
IX	Appendices	325
A	Bregman-Legendre FunctionsA.1Essential Smoothness and Essential Strict ConvexityA.2Bregman Projections onto Closed Convex SetsA.3Bregman-Legendre FunctionsA.4Useful Results about Bregman-Legendre Functions	327 327 328 328 328 329
В	Bregman-Paracontractive OperatorsB.1Bregman ParacontractionsB.2Extending the EKN TheoremB.3Multiple Bregman Distances	331 331 334 335
С	The Fourier TransformC.1Fourier-Transform PairsC.2The Dirac DeltaC.3Practical LimitationsC.4Two-Dimensional Fourier Transforms	337 337 340 341 343
D	The EM AlgorithmD.1The Discrete CaseD.2The Continuous Case	345 345 347
E	Using Prior Knowledge in Remote Sensing E.1 The Optimization Approach E.2 Introduction to Hilbert Space E.3 A Class of Inner Products E.4 Minimum-T-Norm Solutions E.5 The Case of Fourier-Transform Data	349 349 350 352 352 353

xiv		Co	ntents
F	Opti	mization in Remote Sensing	357
	$\mathbf{F.1}$	The General Form of the Cost Function	357
	F.2	The Conditions	358
Bil	oliogr	aphy	361
Ine	dex		373