

Contents

Preface	v
Editor's Preface	vii
I Introduction	1
1 <i>S. I. Kabanikhin</i>	
Inverse Problems of Mathematical Physics	3
1.1 Introduction	3
1.2 Examples of Inverse and Ill-posed Problems	12
1.3 Well-posed and Ill-posed Problems	24
1.4 The Tikhonov Theorem	26
1.5 The Ivanov Theorem: Quasi-solution	29
1.6 The Lavrentiev's Method	33
1.7 The Tikhonov Regularization Method	35
References	44
II Recent Advances in Regularization Theory and Methods	47
2 <i>D. V. Lukyanenko and A. G. Yagola</i>	
Using Parallel Computing for Solving Multidimensional Ill-posed Problems	49
2.1 Introduction	49
2.2 Using Parallel Computing	51
2.2.1 Main idea of parallel computing	51
2.2.2 Parallel computing limitations	52
2.3 Parallelization of Multidimensional Ill-posed Problem	53
2.3.1 Formulation of the problem and method of solution	53
2.3.2 Finite-difference approximation of the functional and its gradient	56
2.3.3 Parallelization of the minimization problem	58
2.4 Some Examples of Calculations	61
2.5 Conclusions	63
References	63

3	<i>M. T. Nair</i>	
	Regularization of Fredholm Integral Equations of the First Kind using Nyström Approximation	65
3.1	Introduction	65
3.2	Nyström Method for Regularized Equations	68
3.2.1	Nyström approximation of integral operators	68
3.2.2	Approximation of regularized equation	69
3.2.3	Solvability of approximate regularized equation	70
3.2.4	Method of numerical solution	73
3.3	Error Estimates	74
3.3.1	Some preparatory results	74
3.3.2	Error estimate with respect to $\ \cdot\ _2$	77
3.3.3	Error estimate with respect to $\ \cdot\ _\infty$	77
3.3.4	A modified method	78
3.4	Conclusion	80
	References	81
4	<i>T. Y. Xiao, H. Zhang and L. L. Hao</i>	
	Regularization of Numerical Differentiation: Methods and Applications	83
4.1	Introduction	83
4.2	Regularizing Schemes	87
4.2.1	Basic settings	87
4.2.2	Regularized difference method (RDM)	88
4.2.3	Smoother-Based regularization (SBR)	89
4.2.4	Mollifier regularization method (MRM)	90
4.2.5	Tikhonov's variational regularization (TiVR)	92
4.2.6	Lavrentiev regularization method (LRM)	93
4.2.7	Discrete regularization method (DRM)	94
4.2.8	Semi-Discrete Tikhonov regularization (SDTR)	96
4.2.9	Total variation regularization (TVR)	99
4.3	Numerical Comparisons	102
4.4	Applied Examples	105
4.4.1	Simple applied problems	106
4.4.2	The inverse heat conduct problems (IHCP)	107
4.4.3	The parameter estimation in new product diffusion model	108
4.4.4	Parameter identification of Sturm-Liouville operator	110
4.4.5	The numerical inversion of Abel transform	112
4.4.6	The linear viscoelastic stress analysis	114
4.5	Discussion and Conclusion	115
	References	117

5	<i>C. L. Fu, H. Cheng and Y. J. Ma</i>	
	Numerical Analytic Continuation and Regularization	121
5.1	Introduction	121
5.2	Description of the Problems in Strip Domain and Some Assumptions	124
5.2.1	Description of the problems	124
5.2.2	Some assumptions	125
5.2.3	The ill-posedness analysis for the Problems 5.2.1 and 5.2.2	125
5.2.4	The basic idea of the regularization for Problems 5.2.1 and 5.2.2	126
5.3	Some Regularization Methods	126
5.3.1	Some methods for solving Problem 5.2.1	126
5.3.2	Some methods for solving Problem 5.2.2	133
5.4	Numerical Tests	135
	References	140
6	<i>G. S. Li</i>	
	An Optimal Perturbation Regularization Algorithm for Function Reconstruction and Its Applications	143
6.1	Introduction	143
6.2	The Optimal Perturbation Regularization Algorithm	144
6.3	Numerical Simulations	147
6.3.1	Inversion of time-dependent reaction coefficient	147
6.3.2	Inversion of space-dependent reaction coefficient	149
6.3.3	Inversion of state-dependent source term	151
6.3.4	Inversion of space-dependent diffusion coefficient	157
6.4	Applications	159
6.4.1	Determining magnitude of pollution source	159
6.4.2	Data reconstruction in an undisturbed soil-column experiment	162
6.5	Conclusions	165
	References	166
7	<i>L. V. Zotov and V. L. Panteleev</i>	
	Filtering and Inverse Problems Solving	169
7.1	Introduction	169
7.2	SLAE Compatibility	170
7.3	Conditionality	171
7.4	Pseudosolutions	173
7.5	Singular Value Decomposition	175
7.6	Geometry of Pseudosolution	177

7.7	Inverse Problems for the Discrete Models of Observations . . .	178
7.8	The Model in Spectral Domain	180
7.9	Regularization of Ill-posed Systems	181
7.10	General Remarks, the Dilemma of Bias and Dispersion	181
7.11	Models, Based on the Integral Equations	184
7.12	Panteleev Corrective Filtering	185
7.13	Philips-Tikhonov Regularization	186
	References	194
III Optimal Inverse Design and Optimization Methods		195
8	<i>G. S. Dulikravich and I. N. Egorov</i> Inverse Design of Alloys' Chemistry for Specified Thermo-Mechanical Properties by using Multi-objective Optimization	197
8.1	Introduction	198
8.2	Multi-Objective Constrained Optimization and Response Surfaces	199
8.3	Summary of IOSO Algorithm	201
8.4	Mathematical Formulations of Objectives and Constraints . . .	203
8.5	Determining Names of Alloying Elements and Their Concentra- tions for Specified Properties of Alloys	212
8.6	Inverse Design of Bulk Metallic Glasses	214
8.7	Open Problems	215
8.8	Conclusions	218
	References	219
9	<i>Z. H. Xiang</i> Two Approaches to Reduce the Parameter Identification Errors	221
9.1	Introduction	221
9.2	The Optimal Sensor Placement Design	223
9.2.1	The well-posedness analysis of the parameter identifica- tion procedure	223
9.2.2	The algorithm for optimal sensor placement design . . .	226
9.2.3	The integrated optimal sensor placement and parameter identification algorithm	229
9.2.4	Examples	229
9.3	The Regularization Method with the Adaptive Updating of A- priori Information	233

9.3.1	Modified extended Bayesian method for parameter identification	234
9.3.2	The well-posedness analysis of modified extended Bayesian method	234
9.3.3	Examples	236
9.4	Conclusion	238
	References	238
10	<i>Y. H. Dai</i> A General Convergence Result for the BFGS Method	241
10.1	Introduction	241
10.2	The BFGS Algorithm	243
10.3	A General Convergence Result for the BFGS Algorithm	244
10.4	Conclusion and Discussions	246
	References	247
IV Recent Advances in Inverse Scattering		249
11	<i>X. D. Liu and B. Zhang</i> Uniqueness Results for Inverse Scattering Problems	251
11.1	Introduction	251
11.2	Uniqueness for Inhomogeneity n	256
11.3	Uniqueness for Smooth Obstacles	256
11.4	Uniqueness for Polygon or Polyhedra	262
11.5	Uniqueness for Balls or Discs	263
11.6	Uniqueness for Surfaces or Curves	265
11.7	Uniqueness Results in a Layered Medium	265
11.8	Open Problems	272
	References	276
12	<i>G. Bao and P. J. Li</i> Shape Reconstruction of Inverse Medium Scattering for the Helmholtz Equation	283
12.1	Introduction	283
12.2	Analysis of the scattering map	285
12.3	Inverse medium scattering	290
12.3.1	Shape reconstruction	291
12.3.2	Born approximation	292
12.3.3	Recursive linearization	294
12.4	Numerical experiments	298
12.5	Concluding remarks	303
	References	303

V Inverse Vibration, Data Processing and Imaging 307

13 *G. M. Kuramshina, I. V. Kochikov and A. V. Stepanova*

Numerical Aspects of the Calculation of Molecular Force Fields from Experimental Data 309

13.1 Introduction	309
13.2 Molecular Force Field Models	311
13.3 Formulation of Inverse Vibration Problem	312
13.4 Constraints on the Values of Force Constants Based on Quantum Mechanical Calculations	314
13.5 Generalized Inverse Structural Problem	319
13.6 Computer Implementation	321
13.7 Applications	323
References	327

14 *J. J. Liu and H. L. Xu*

Some Mathematical Problems in Biomedical Imaging 331

14.1 Introduction	331
14.2 Mathematical Models	334
14.2.1 Forward problem	334
14.2.2 Inverse problem	336
14.3 Harmonic B_z Algorithm	339
14.3.1 Algorithm description	340
14.3.2 Convergence analysis	342
14.3.3 The stable computation of ΔB_z	344
14.4 Integral Equations Method	348
14.4.1 Algorithm description	348
14.4.2 Regularization and discretization	352
14.5 Numerical Experiments	354
References	362

VI Numerical Inversion in Geosciences 367

15 *S. I. Kabanikhin and M. A. Shishlenin*

Numerical Methods for Solving Inverse Hyperbolic Problems 369

15.1 Introduction	369
15.2 Gel'fand-Levitan-Krein Method	370
15.2.1 The two-dimensional analogy of Gel'fand-Levitan-Krein equation	374
15.2.2 N -approximation of Gel'fand-Levitan-Krein equation	377

15.2.3 Numerical results and remarks	379
15.3 Linearized Multidimensional Inverse Problem for the Wave Equation	379
15.3.1 Problem formulation	381
15.3.2 Linearization	382
15.4 Modified Landweber Iteration	384
15.4.1 Statement of the problem	385
15.4.2 Landweber iteration	387
15.4.3 Modification of algorithm	388
15.4.4 Numerical results	389
References	390

16 *H. B. Song, X. H. Huang, L. M. Pinheiro, Y. Song, C. Z. Dong and Y. Bai*

Inversion Studies in Seismic Oceanography 395

16.1 Introduction of Seismic Oceanography	395
16.2 Thermohaline Structure Inversion	398
16.2.1 Inversion method for temperature and salinity	398
16.2.2 Inversion experiment of synthetic seismic data	399
16.2.3 Inversion experiment of GO data (Huang et al., 2011)	402
16.3 Discussion and Conclusion	406
References	408

17 *L. J. Gelius*

Image Resolution Beyond the Classical Limit 411

17.1 Introduction	411
17.2 Aperture and Resolution Functions	412
17.3 Deconvolution Approach to Improved Resolution	417
17.4 MUSIC Pseudo-Spectrum Approach to Improved Resolution	424
17.5 Concluding Remarks	434
References	436

18 *Y. F. Wang, Z. H. Li and C. C. Yang*

Seismic Migration and Inversion 439

18.1 Introduction	439
18.2 Migration Methods: A Brief Review	440
18.2.1 Kirchhoff migration	440
18.2.2 Wave field extrapolation	441
18.2.3 Finite difference migration in $\omega - X$ domain	442
18.2.4 Phase shift migration	443
18.2.5 Stolt migration	443
18.2.6 Reverse time migration	446

18.2.7	Gaussian beam migration	447
18.2.8	Interferometric migration	447
18.2.9	Ray tracing	449
18.3	Seismic Migration and Inversion	452
18.3.1	The forward model	454
18.3.2	Migration deconvolution	456
18.3.3	Regularization model	457
18.3.4	Solving methods based on optimization	458
18.3.5	Preconditioning	462
18.3.6	Preconditioners	464
18.4	Illustrative Examples	465
18.4.1	Regularized migration inversion for point diffraction scatterers	465
18.4.2	Comparison with the interferometric migration	468
18.5	Conclusion	468
	References	471
19	<i>Y. F. Wang, J. J. Cao, T. Sun and C. C. Yang</i>	
	Seismic Wavefields Interpolation Based on Sparse Regularization and Compressive Sensing	475
19.1	Introduction	475
19.2	Sparse Transforms	477
19.2.1	Fourier, wavelet, Radon and ridgelet transforms	477
19.2.2	The curvelet transform	480
19.3	Sparse Regularizing Modeling	481
19.3.1	Minimization in l_0 space	481
19.3.2	Minimization in l_1 space	481
19.3.3	Minimization in l_p - l_q space	482
19.4	Brief Review of Previous Methods in Mathematics	482
19.5	Sparse Optimization Methods	485
19.5.1	l_0 quasi-norm approximation method	485
19.5.2	l_1 -norm approximation method	487
19.5.3	Linear programming method	489
19.5.4	Alternating direction method	491
19.5.5	l_1 -norm constrained trust region method	493
19.6	Sampling	496
19.7	Numerical Experiments	497
19.7.1	Reconstruction of shot gathers	497
19.7.2	Field data	498
19.8	Conclusion	503
	References	503

20	<i>H. Yang</i>	
	Some Researches on Quantitative Remote Sensing Inversion	509
20.1	Introduction	509
20.2	Models	511
20.3	A Priori Knowledge	514
20.4	Optimization Algorithms	516
20.5	Multi-stage Inversion Strategy	520
20.6	Conclusion	524
	References	525
	Index	529