

Contents

Preface	vii
Preface to Volume 2	ix
Contents of Volume 2	xvi
List of Main Notation	xviii

Chapter 1. Basic Concepts	1
1.1. The Nature of Spectral Analysis	1
1.2. Types of Processes	2
1.3. Periodic and Non-periodic Functions	3
1.4. Generalized Harmonic Analysis	7
1.5. Energy Distribution	8
1.6. Random Processes	10
1.7. Stationary Random Processes	14
1.8. Spectral Analysis of Stationary Random Processes	15
1.9. Spectral Analysis of Non-stationary Random Processes	17
1.10. Time Series Analysis: Use of Spectral Analysis in Practice	18

Chapter 2. Elements of Probability Theory	28
2.1. Introduction	28
2.2. Some Terminology	28
2.3. Definition of Probability	30
2.3.1. Axiomatic approach to probability	31
2.3.2. The classical definition	33
2.3.3. Probability spaces	34
2.4. Conditional Probability and Independence	34
2.4.1. Independent events	36
2.5. Random Variables	37
2.5.1. Defining probabilities for random variables	38
2.6. Distribution Functions	39
2.6.1. Decomposition of distribution functions	40

2.7. Discrete, Continuous and Mixed Distributions	41
2.8. Means, Variances and Moments	47
2.8.1. Chebyshev's inequality	51
2.9. The Expectation Operator	52
2.9.1. General transformations	53
2.10. Generating Functions	55
2.10.1. Probability generating functions	55
2.10.2. Moment generating functions	56
2.10.3. Characteristic functions	57
2.10.4. Cumulant generating functions	58
2.11. Some Special Distributions	59
2.12. Bivariate Distributions	67
2.12.1. Bivariate cumulative distribution functions	68
2.12.2. Marginal distributions	69
2.12.3. Conditional distributions	70
2.12.4. Independent random variables	71
2.12.5. Expectation for bivariate distributions	72
2.12.6. Conditional expectation	73
2.12.7. Bivariate moments	77
2.12.8. Bivariate moment generating functions and characteristic functions	82
2.12.9. Bivariate normal distribution	84
2.12.10. Transformations of variables	86
2.13. Multivariate Distributions	87
2.13.1. Mean and variance of linear combinations	89
2.13.2. Multivariate normal distribution	90
2.14. The Law of Large Numbers and the Central Limit Theorem	92
2.14.1. The law of large numbers	92
2.14.2. Application to the binomial distribution	94
2.14.3. The central limit theorem	95
2.14.4. Properties of means and variances of random samples	96

Chapter 3. Stationary Random Processes

3.1. Probabilistic Description of Random Processes	100
3.1.1. Realizations and ensembles	101
3.1.2. Specification of a random process	101
3.2. Stationary Processes	104
3.3. The Autocovariance and Autocorrelation Functions	106
3.3.1. General properties of $R(\tau)$ and $\rho(\tau)$	108
3.3.2. Positive semi-definite property of $R(\tau)$ and $\rho(\tau)$	109
3.3.3. Complex valued processes	110
3.3.4. Use of the autocorrelation function process in practice	111
3.4. Stationary and Evolutionary Processes	112
3.4.1. Gaussian (normal) processes	113
3.5. Special Discrete Parameter Models	114

3.5.1. Purely random processes: "white noise"	114
3.5.2. First order autoregressive processes (linear Markov processes)	116
3.5.3. Second order autoregressive processes	123
3.5.4. Autoregressive processes of general order	132
3.5.5. Moving average processes	135
3.5.6. Mixed autoregressive/moving average processes	138
3.5.7. The general linear process	141
3.5.8. Harmonic processes	147
3.6. Stochastic Limiting Operations	150
3.6.1. Stochastic continuity	151
3.6.2. Stochastic differentiability	153
3.6.3. Integration	154
3.6.4. Interpretation of the derivatives of the auto-correlation function	155
3.7. Standard Continuous Parameter Models	156
3.7.1. Purely random processes (continuous "white noise")	156
3.7.2. First order autoregressive processes	158
3.7.3. Examples of first order autoregressive processes	167
3.7.4. Second order autoregressive processes	169
3.7.5. Autogressive processes of general order	174
3.7.6. Moving average processes	174
3.7.7. Mixed autoregressive/moving average processes	176
3.7.8. General linear processes	177
3.7.9. Harmonic processes	179
3.7.10. Filtered Poisson process, shot noise and Campbell's theorem	179

Chapter 4. Spectral Analysis

4.1. Introduction	184
4.2. Fourier Series for Functions with Periodicity 2π	184
4.2.1. The L^2 theory for Fourier series	189
4.2.2. Geometrical interpretation of Fourier series: Hilbert space	190
4.3. Fourier Series for Functions of General Periodicity	194
4.4. Spectral Analysis of Periodic Functions	194
4.4.1. Functions of general periodicity	197
4.5. Non-periodic Functions: Fourier Integral	198
4.5.1. Nature of conditions for the existence of Fourier series and Fourier integrals	202
4.6. Spectral Analysis of Non-periodic Functions	204
4.7. Spectral Analysis of Stationary Processes	206
4.8. Relationship between the Spectral Density Function and the Autocovariance and Autocorrelation Functions	210

4.8.1. Normalized power spectra	215
4.8.2. The Wiener-Khintchine theorem	218
4.8.3. Discrete parameter processes	222
4.9. Decomposition of the Integrated Spectrum	226
4.10. Examples of Spectra for some Simple Models	233
4.11. Spectral Representation of some Stationary Processes	243
4.12. Linear Transformations and Filters	263
4.12.1. Filter terminology: gain and phase	270
4.12.2. Operational forms of filter relationships: calculation of spectra	276
4.12.3. Transformation of the autocovariance function	280
4.12.4. Processes with rational spectra	283
4.12.5. Axiomatic treatment of filters	285

Chapter 5. Estimation in the Time Domain	291
5.1. Time Series Analysis	291
5.2. Basic Ideas of Statistical Inference	292
5.2.1. Point estimation	295
5.2.2. General methods of estimation	304
5.3. Estimation of Autocovariance and Autocorrelation Functions	317
5.3.1. Form of the data	317
5.3.2. Estimation of the mean	318
5.3.3. Estimation of the autocovariance function	321
5.3.4. Estimation of the autocorrelation function	330
5.3.5. Asymptotic distribution of the sample mean autocovariances and autocorrelations	337
5.3.6. The ergodic property	340
5.3.7. Continuous parameter processes	343
5.4. Estimation of Parameters in Standard Models	345
5.4.1. Estimation of parameters in autoregressive models	346
5.4.2. Estimation of parameters in moving average models	354
5.4.3. Estimation of parameters in mixed ARMA models	359
5.4.4. Confidence intervals for the parameters	364
5.4.5. Determining the order of the model	370
5.4.6. Continuous parameter models	380
5.5. Analysis of the Canadian Lynx Data	384

Chapter 6. Estimation in the Frequency Domain	389
6.1. Discrete spectra	390
6.1.1. Estimation	391
6.1.2. Periodogram analysis	394
6.1.3. Sampling properties of the periodogram	397
6.1.4. Tests for periodogram ordinates	406

6.2. Continuous Spectra	415
6.2.1. Finite Fourier transforms	418
6.2.2. Properties of the periodogram of a linear process	420
6.2.3. Consistent estimates of the spectral density function: spectral windows	432
6.2.4. Sampling properties of spectral estimates	449
6.2.5. Estimation of the integrated spectrum	471
6.2.6. Goodness-of-fit tests	475
6.2.7. Continuous parameter processes	494
6.3. Mixed Spectra	449

Chapter 7. Spectral Analysis in Practice

7.1. Setting up a Spectral Analysis	502
7.1.1. The aliasing effect	504
7.2. Measures of Precision of Spectral Estimates	570
7.3. Resolvability and Bandwidth	513
7.3.1. Role of spectral bandwidth	513
7.3.2. Role of window bandwidth	517
7.4. Design Relations for Spectral Estimation: Choice of Window Parameters, Record Length and Frequency Interval	528
7.4.1. Prewhitenning and tapering	556
7.5. Choice of Window	563
7.6. Computation of Spectral Estimates: The Fast Fourier Transform	575
7.7. Trend Removal and Seasonal Adjustment: Regression Analysis and Digital Filters	587
7.8. Autoregressive, ARMA, and Maximum Entropy Spectral Estimation; CAT Criterion	600
7.9. Some Examples	607

Chapter 8. Analysis of Processes with Mixed Spectra

8.1. Nature of the Problem	613
8.2. Types of Models	614
8.3. Tests Based on the Periodogram	616
8.4. The $P(\lambda)$ Test	626
8.5. Analysis of Simulated Series	642
8.6. Estimation of the Spectral Density Function	648

Appendix	Ai
References	Ri
Author Index	Ii
Subject Index	Ivii