Contents

1	An In	vitation to Compressive Sensing	1
	1.1	What is Compressive Sensing?	1
	1.2	Applications, Motivations, and Extensions	8
	1.3	Overview of the Book	23
	Notes		33
2	Spars	e Solutions of Underdetermined Systems	41
	2.1	Sparsity and Compressibility	41
	2.2	Minimal Number of Measurements	48
	2.3	NP-Hardness of ℓ_0 -Minimization	53
	Notes		56
	Exerc	ises	57
3	Basic	Algorithms	61
	3.1	Optimization Methods	61
	3.2	Greedy Methods	65
	3.3	Thresholding-Based Methods	69
	Notes	-	71
	Exerc	ises	73
4	Basis	Pursuit	77
	4.1	Null Space Property	78
	4.2	Stability	82
	4.3	Robustness	85
	4.4	Recovery of Individual Vectors	90
	4.5	The Projected Cross-Polytope	98
	4.6	Low-Rank Matrix Recovery	102
	Notes	·	104
	Exerc	ises	105
5	Cohe	rence	111
	5.1	Definitions and Basic Properties	111
	5.2	Matrices with Small Coherence	113

	5.3	Analysis of Orthogonal Matching Pursuit	123
	5.4	Analysis of Basis Pursuit	124 126
	5.5	Analysis of Thresholding Algorithms	120
		ses	128
			127
6	Restri	icted Isometry Property	133
	6.1	Definitions and Basic Properties	133
	6.2	Analysis of Basis Pursuit	141
	6.3	Analysis of Thresholding Algorithms	148
	6.4	Analysis of Greedy Algorithms	156
			168
	Exerci	ses	170
7	Basic	Tools from Probability Theory	175
	7.1	Essentials from Probability	175
	7.2	Moments and Tails	185
	7.3	Cramér's Theorem and Hoeffding's Inequality	188
	7.4	Subgaussian Random Variables	191
	7.5	Bernstein Inequalities	196
		•	198
	Exerci	ses	199
0	A deco	and Table from Drobability Theory	201
8	Auva	nced Tools from Probability Theory	201
	0.1	Expectation of Norma of Coussian Vectors	202
	8.1	Expectation of Norms of Gaussian Vectors	202
	8.2	Rademacher Sums and Symmetrization	205
	8.2 8.3	Rademacher Sums and Symmetrization	205 206
	8.2 8.3 8.4	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling	205 206 211
	8.2 8.3 8.4 8.5	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality	205 206 211 217
	8.2 8.3 8.4 8.5 8.6	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality	205 206 211 217 224
	8.2 8.3 8.4 8.5 8.6 8.7	Rademacher Sums and Symmetrization	205 206 211 217 224 227
	8.2 8.3 8.4 8.5 8.6 8.7 8.8	Rademacher Sums and Symmetrization	205 206 211 217 224 227 237
	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9	Rademacher Sums and SymmetrizationKhintchine InequalitiesDecouplingNoncommutative Bernstein InequalityDudley's InequalitySlepian's and Gordon's LemmasConcentration of MeasureBernstein Inequality for Suprema of Empirical Processes	205 206 211 217 224 227 237 247
	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes	Rademacher Sums and Symmetrization	205 206 211 217 224 227 237 247 260
	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerct	Rademacher Sums and Symmetrization	205 206 211 217 224 227 237 247 260 265
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices	205 206 211 217 224 227 237 247 260 265 271
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices	205 206 211 217 224 227 237 247 260 265 271 272
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1 9.2	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices Nonuniform Recovery	205 206 211 217 224 227 237 247 260 265 271 272 281
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1 9.2 9.3	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices Nonuniform Recovery Restricted Isometry Property for Gaussian Matrices	205 206 211 217 224 227 237 247 260 265 271 272 281 291
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1 9.2 9.3 9.4	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices Nonuniform Recovery Restricted Isometry Property for Gaussian Matrices Null Space Property for Gaussian Matrices	205 206 211 217 224 227 237 247 260 265 271 272 281 291 293
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1 9.2 9.3 9.4 9.5	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices Nonuniform Recovery Restricted Isometry Property for Gaussian Matrices Null Space Property for Gaussian Matrices Relation to Johnson–Lindenstrauss Embeddings	205 206 211 217 224 227 237 247 260 265 271 272 281 291 293 298
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1 9.2 9.3 9.4 9.5 Notes	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices Nonuniform Recovery Restricted Isometry Property for Gaussian Matrices Null Space Property for Gaussian Matrices Relation to Johnson–Lindenstrauss Embeddings	205 206 211 217 224 227 237 247 260 265 271 272 281 293 298 302
9	8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 Notes Exerci Spars 9.1 9.2 9.3 9.4 9.5 Notes	Rademacher Sums and Symmetrization Khintchine Inequalities Decoupling Noncommutative Bernstein Inequality Dudley's Inequality Slepian's and Gordon's Lemmas Concentration of Measure Bernstein Inequality for Suprema of Empirical Processes ises e Recovery with Random Matrices Restricted Isometry Property for Subgaussian Matrices Nonuniform Recovery Restricted Isometry Property for Gaussian Matrices Null Space Property for Gaussian Matrices Relation to Johnson–Lindenstrauss Embeddings	205 206 211 217 224 227 237 247 260 265 271 272 281 291 293 298

10	Gelfar	nd Widths of ℓ_1 -Balls	311
	10.1	Definitions and Relation to Compressive Sensing	311
	10.2	Estimate for the Gelfand Widths of ℓ_1 -Balls	317
	10.3	Applications to the Geometry of Banach Spaces	322
	Notes.		327
	Exerci	ses	328
11	Instan	ce Optimality and Quotient Property	331
_	11.1	Uniform Instance Optimality	332
	11.2	Robustness and Quotient Property	337
	11.3	Quotient Property for Random Matrices	343
	11.4	Nonuniform Instance Optimality	358
	Notes.		362
	Exerci	ses	363
12	Rando	om Sampling in Bounded Orthonormal Systems	367
	12.1	Bounded Orthonormal Systems	368
	12.2	Uncertainty Principles and Lower Bounds	374
	12.3	Nonuniform Recovery: Random Sign Patterns	384
	12.4	Nonuniform Recovery: Deterministic Sign Patterns	392
	12.5	Restricted Isometry Property	404
	12.6	Discrete Bounded Orthonormal Systems	416
	12.7	Relation to the Λ_1 -Problem	418
	Notes.		420
	Exerci	ses	431
13	Lossle	ss Expanders in Compressive Sensing	435
	13.1	Definitions and Basic Properties	435
	13.2	Existence of Lossless Expanders	439
	13.3	Analysis of Basis Pursuit	443
	13.4	Analysis of an Iterative Thresholding Algorithm	447
	13.5	Analysis of a Simple Sublinear-Time Algorithm	452
	Notes.		455
	Exerci	ses	456
14	Recov	ery of Random Signals using Deterministic Matrices	459
	14.1	Conditioning of Random Submatrices	460
	14.2	Sparse Recovery via ℓ_1 -Minimization	469
	Notes.		471
	Exerci	ses	473
15	Algori	ithms for ℓ_1 -Minimization	475
	15.1	The Homotopy Method	476
	15.2	Chambolle and Pock's Primal-Dual Algorithm	481
	15.3	Iteratively Reweighted Least Squares	492
		,	503
		ses	511

Appendix			
Α	Matrix Analysis		
A	A.1	Vector and Matrix Norms	515 515
	A.1 A.2	The Singular Value Decomposition	525
	A.2 A.3	Least Squares Problems	531
	A.4	Vandermonde Matrices	535
	A.5	Matrix Functions	537
В	Conv	ex Analysis	543
	B.1	Convex Sets	543
	B.2	Convex Functions	545
	B.3	The Convex Conjugate	549
	B.4	The Subdifferential	551
	B.5	Convex Optimization Problems	555
	B.6	Matrix Convexity	564
С	Misce	llanea	573
	C.1	Fourier Analysis	573
	C.2	Covering Numbers	575
	C.3	The Gamma Function and Stirling's Formula	577
	C.4	The Multinomial Theorem	580
	C.5	Some Elementary Estimates	580
	C.6	Estimates of Some Integrals	581
	C.7	Hahn–Banach Theorems	583
	C.8	Smoothing Lipschitz Functions	583
	C.9	Weak and Distributional Derivatives	585
	C.10	Differential Inequalities	587
Lis	t of Syı	nbols	589
Re	References		
Index			