

Contents

Part I Introduction

1	Inverse Problems, Optimization and Regularization: A Multi-Disciplinary Subject	3
	<i>Yanfei Wang and Changchun Yang</i>	
1.1	Introduction	3
1.2	Examples about mathematical inverse problems	4
1.3	Examples in applied science and engineering	5
1.4	Basic theory	12
1.5	Scientific computing	12
1.6	Conclusion	13
	References	13

Part II Regularization Theory and Recent Developments

2	Ill-Posed Problems and Methods for Their Numerical Solution 17	
	<i>Anatoly G. Yagola</i>	
2.1	Well-posed and ill-posed problems	18
2.2	Definition of the regularizing algorithm	22
2.3	Ill-posed problems on compact sets	25
2.4	Ill-posed problems with sourcewise represented solutions	27
2.5	Variational approach for constructing regularizing algorithms	28
2.6	Nonlinear ill-posed problems	32
2.7	Iterative and other methods	33
	References	34
3	Inverse Problems with <i>A Priori</i> Information	35
	<i>Vladimir V. Vasin</i>	
3.1	Introduction	35

3.2	Formulation of the problem with <i>a priori</i> information	39
3.3	The main classes of mappings of the Fejér type and their properties	41
3.4	Convergence theorems of the method of successive approximations for the pseudo-contractive operators	46
3.5	Examples of operators of the Fejér type	50
3.6	Fejér processes for nonlinear equations	53
3.7	Applied problems with <i>a priori</i> information and methods for solution	57
3.7.1	Atomic structure characterization	57
3.7.2	Radiolocation of the ionosphere	58
3.7.3	Image reconstruction	59
3.7.4	Thermal sounding of the atmosphere	60
3.7.5	Testing a wellbore/reservoir	61
3.8	Conclusions	62
	References	62
4	Regularization of Naturally Linearized Parameter Identification Problems and the Application of the Balancing Principle	65
	<i>Hui Cao and Sergei Pereverzyev</i>	
4.1	Introduction	65
4.2	Discretized Tikhonov regularization and estimation of accuracy	68
4.2.1	Generalized source condition	68
4.2.2	Discretized Tikhonov regularization	70
4.2.3	Operator monotone index functions	71
4.2.4	Estimation of the accuracy	73
4.3	Parameter identification in elliptic equation	75
4.3.1	Natural linearization	75
4.3.2	Data smoothing and noise level analysis	77
4.3.3	Estimation of the accuracy	78
4.3.4	Balancing principle	80
4.3.5	Numerical examples	83
4.4	Parameter identification in parabolic equation	85
4.4.1	Natural linearization for recovering $b(x) = a(u(T, x))$	86
4.4.2	Regularized identification of the diffusion coefficient $a(u)$	89
4.4.3	Extended balancing principle	92
4.4.4	Numerical examples	99
	References	103
5	Extrapolation Techniques of Tikhonov Regularization	107
	<i>Tingyan Xiao, Yuan Zhao and Guozhong Su</i>	
5.1	Introduction	107
5.2	Notations and preliminaries	109

5.3	Extrapolated regularization based on vector-valued function approximation	111
5.3.1	Extrapolated scheme based on Lagrange interpolation	112
5.3.2	Extrapolated scheme based on Hermitian interpolation	114
5.3.3	Extrapolation scheme based on rational interpolation	116
5.4	Extrapolated regularization based on improvement of regularizing qualification	118
5.5	The choice of parameters in the extrapolated regularizing approximation	119
5.6	Numerical experiments	122
5.7	Conclusion	125
	References	126
6	Modified Regularization Scheme with Application in Reconstructing Neumann-Dirichlet Mapping	127
	<i>Pingli Xie and Jin Cheng</i>	
6.1	Introduction	127
6.2	Regularization method	129
6.3	Computational aspect	131
6.4	Numerical simulation results for the modified regularization	131
6.5	The Neumann-Dirichlet mapping for elliptic equation of second order	135
6.6	The numerical results of the Neumann-Dirichlet mapping	136
6.7	Conclusion	138
	References	138
	Part III Nonstandard Regularization and Advanced Optimization Theory and Methods	
7	Gradient Methods for Large Scale Convex Quadratic Functions	141
	<i>Yaxiang Yuan</i>	
7.1	Introduction	141
7.2	A generalized convergence result	143
7.3	Short BB steps	147
7.4	Numerical results	149
7.5	Discussion and conclusion	154
	References	155
8	Convergence Analysis of Nonlinear Conjugate Gradient Methods	157
	<i>Yuhong Dai</i>	
8.1	Introduction	157
8.2	Some preliminaries	160
8.3	A sufficient and necessary condition on β_k	161

8.3.1	Proposition of the condition	161
8.3.2	Sufficiency of (8.3.5)	163
8.3.3	Necessity of (8.3.5)	166
8.4	Applications of the condition (8.3.5)	168
8.4.1	Property (#)	168
8.4.2	Applications to some known conjugate gradient methods	170
8.4.3	Application to a new conjugate gradient method	175
8.5	Discussion	178
	References	180
9	Full Space and Subspace Methods for Large Scale Image Restoration	183
	<i>Yanfei Wang, Shiqian Ma and Qinghua Ma</i>	
9.1	Introduction	183
9.2	Image restoration without regularization	185
9.3	Image restoration with regularization	186
9.4	Optimization methods for solving the smoothing regularized functional	187
9.4.1	Minimization of the convex quadratic programming problem with projection	187
9.4.2	Limited memory BFGS method with projection	188
9.4.3	Subspace trust region methods	191
9.5	Matrix-Vector Multiplication (MVM)	193
9.5.1	MVM: FFT-based method	193
9.5.2	MVM with sparse matrix	194
9.6	Numerical experiments	197
9.7	Conclusions	200
	References	200
	Part IV Numerical Inversion in Geoscience and Quantitative Remote Sensing	
10	Some Reconstruction Methods for Inverse Scattering Problems	205
	<i>Jijun Liu and Haibing Wang</i>	
10.1	Introduction	206
10.2	Iterative methods and decomposition methods	210
10.2.1	Iterative methods	210
10.2.2	Decomposition methods	212
10.2.3	Hybrid method	217
10.3	Singular source methods	218
10.3.1	Probe method	218
10.3.2	Singular sources method	220
10.3.3	Linear sampling method	227

10.3.4	Factorization method	228
10.3.5	Range test method	231
10.3.6	No response test method	233
10.4	Numerical schemes	235
	References	244
11	Inverse Problems of Molecular Spectra Data Processing	249
	<i>Gulnara Kuramshina</i>	
11.1	Introduction	249
11.2	Inverse vibrational problem	250
11.3	The mathematical formulation of the inverse vibrational problem	253
11.4	Regularizing algorithms for solving the inverse vibrational problem	255
11.5	Model of scaled molecular force field	259
11.6	General inverse problem of structural chemistry	261
11.7	Intermolecular potential	265
11.8	Examples of calculations	266
11.8.1	Calculation of methane intermolecular potential	266
11.8.2	Prediction of vibrational spectrum of fullerene C_{240}	267
	References	271
12	Numerical Inversion Methods in Geoscience and Quantitative Remote Sensing	273
	<i>Yanfei Wang and Xiaowen Li</i>	
12.1	Introduction	274
12.2	Examples of quantitative remote sensing inverse problems: land surface parameter retrieval problem	275
12.3	Formulation of the forward and inverse problem	277
12.4	What causes ill-posedness	278
12.5	Tikhonov variational regularization	279
12.5.1	Choices of the scale operator D	279
12.5.2	Regularization parameter selection methods	281
12.6	Solution methods	282
12.6.1	Gradient-type methods	282
12.6.2	Newton-type methods	286
12.7	Numerical examples	292
12.8	Conclusions	297
	References	297
13	Pseudo-Differential Operator and Inverse Scattering of Multidimensional Wave Equation	301
	<i>Hong Liu, Li He</i>	
13.1	Introduction	302
13.2	Notations of operators and symbols	303
13.3	Description in symbol domain	305

13.4 Lie algebra integral expressions	307
13.5 Wave equation on the ray coordinates	308
13.6 Symbol expression of one-way wave operator equations	310
13.7 Lie algebra expression of travel time	312
13.8 Lie algebra integral expression of prediction operator	316
13.9 Spectral factorization expressions of reflection data	319
13.10 Conclusions	323
References	323
14 Tikhonov Regularization for Gravitational Lensing Research.	327
<i>Boris Artamonov, Ekaterina Koptelova, Elena Shimanovskaya and Anatoly G. Yagola</i>	
14.1 Introduction	328
14.2 Regularized deconvolution of images with point sources and smooth background	330
14.2.1 Formulation of the problem	330
14.2.2 Tikhonov regularization approach	333
14.2.3 <i>A priori</i> information	335
14.3 Application of the Tikhonov regularization approach to quasar profile reconstruction	341
14.3.1 Brief introduction to microlensing	341
14.3.2 Formulation of the problem	342
14.3.3 Implementation of the Tikhonov regularization approach .	343
14.3.4 Numerical results of the Q2237 profile reconstruction	345
14.4 Conclusions	345
References	346
Index	349