

# CONTENTS

## CHAPTER I

### Basic Concepts

1. Definition and construction of Lie and associative algebras . . . . .	2
2. Algebras of linear transformations. Derivations . . . . .	5
3. Inner derivations of associative and Lie algebras . . . . .	9
4. Determination of the Lie algebras of low dimensionalities .	11
5. Representations and modules . . . . .	14
6. Some basic module operations . . . . .	19
7. Ideals, solvability, nilpotency . . . . .	23
8. Extension of the base field . . . . .	26

## CHAPTER II

### Solvable and Nilpotent Lie Algebras

1. Weakly closed subsets of an associative algebra . . . . .	31
2. Nil weakly closed sets . . . . .	33
3. Engel's theorem . . . . .	36
4. Primary components. Weight spaces . . . . .	37
5. Lie algebras with semi-simple enveloping associative algebras . . . . .	43
6. Lie's theorems . . . . .	48
7. Applications to abstract Lie algebras. Some counter examples . . . . .	51

## CHAPTER III

### Cartan's Criterion and Its Consequences

1. Cartan subalgebras . . . . .	57
2. Products of weight spaces . . . . .	61
3. An example . . . . .	64
4. Cartan's criteria . . . . .	66
5. Structure of semi-simple algebras . . . . .	70
6. Derivations . . . . .	73
7. Complete reducibility of the representations of semi-simple algebra . . . . .	75

8. Representations of the split three-dimensional simple Lie algebra . . . . .	83
9. The theorems of Levi and Malcev-Harish-Chandra . . . . .	86
10. Cohomology groups of a Lie algebra . . . . .	93
11. More on complete reducibility . . . . .	96

## CHAPTER IV

**Split Semi-simple Lie Algebras**

1. Properties of roots and root spaces . . . . .	108
2. A basic theorem on representations and its consequences for the structure theory . . . . .	112
3. Simple systems of roots . . . . .	119
4. The isomorphism theorem. Simplicity . . . . .	127
5. The determination of the Cartan matrices . . . . .	128
6. Construction of the algebras . . . . .	135
7. Compact forms . . . . .	146

## CHAPTER V

**Universal Enveloping Algebras**

1. Definition and basic properties . . . . .	151
2. The Poincaré-Birkhoff-Witt theorem . . . . .	156
3. Filtration and graded algebra . . . . .	163
4. Free Lie algebras . . . . .	167
5. The Campbell-Hausdorff formula . . . . .	170
6. Cohomology of Lie algebras. The standard complex . . . . .	174
7. Restricted Lie algebras of characteristic $p$ . . . . .	185
8. Abelian restricted Lie algebras . . . . .	192

## CHAPTER VI

**The Theorem of Ado-Iwasawa**

1. Preliminary results . . . . .	199
2. The characteristic zero case . . . . .	201
3. The characteristic $p$ case . . . . .	203

## CHAPTER VII

**Classification of Irreducible Modules**

1. Definition of certain Lie algebras . . . . .	207
---	-----

2. On certain cyclic modules for $\tilde{\mathfrak{g}}$ . . . . .	212
3. Finite-dimensional irreducible modules . . . . .	215
4. Existence theorem and isomorphism theorem for semi-simple Lie algebras . . . . .	220
5. Existence of $E_7$ and $E_8$ . . . . .	223
6. Basic irreducible modules . . . . .	225

## CHAPTER VIII

**Characters of the Irreducible Modules**

1. Some properties of the Weyl group . . . . .	240
2. Freudenthal's formula . . . . .	243
3. Weyl's character formula . . . . .	249
4. Some examples . . . . .	257
5. Applications and further results . . . . .	259

## CHAPTER IX

**Automorphisms**

1. Lemmas from algebraic geometry . . . . .	266
2. Conjugacy of Cartan subalgebras . . . . .	271
3. Non-isomorphism of the split simple Lie algebras . . . . .	274
4. Automorphisms of semi-simple Lie algebras over an algebraically closed field . . . . .	275
5. Explicit determination of the automorphisms for the simple Lie algebras . . . . .	281

## CHAPTER X

**Simple Lie Algebras over an Arbitrary Field**

1. Multiplication algebra and centroid of a non-associative algebra . . . . .	290
2. Isomorphism of extension algebras . . . . .	295
3. Simple Lie algebras of types A-D . . . . .	298
4. Conditions for isomorphism . . . . .	303
5. Completeness theorems . . . . .	308
6. A closer look at the isomorphism conditions . . . . .	311
7. Central simple real Lie algebras . . . . .	313
Bibliography . . . . .	319
Index . . . . .	329