

CONTENTS

List of Figures	x
List of Tables	x
Preface	xi
Standard Notation	xv
I. Overview	3
II. Curves in Projective Space	
1. Projective Space	19
2. Curves and Tangents	24
3. Flexes	32
4. Application to Cubics	40
5. Bezout's Theorem and Resultants	44
III. Cubic Curves in Weierstrass Form	
1. Examples	50
2. Weierstrass Form, Discriminant, j -invariant	56
3. Group Law	67
4. Computations with the Group Law	74
5. Singular Points	77
IV. Mordell's Theorem	
1. Descent	80
2. Condition for Divisibility by 2	85
3. $E(\mathbb{Q})/2E(\mathbb{Q})$, Special Case	88
4. $E(\mathbb{Q})/2E(\mathbb{Q})$, General Case	92
5. Height and Mordell's Theorem	95
6. Geometric Formula for Rank	102
7. Upper Bound on the Rank	107
8. Construction of Points in $E(\mathbb{Q})$	115
9. Appendix on Algebraic Number Theory	122
V. Torsion Subgroup of $E(\mathbb{Q})$	
1. Overview	130
2. Reduction Modulo p	134
3. p -adic Filtration	137
4. Lutz-Nagell Theorem	144
5. Construction of Curves with Prescribed Torsion	145
6. Torsion Groups for Special Curves	148

VI. Complex Points	
1. Overview	151
2. Elliptic Functions	152
3. Weierstrass \wp Function	153
4. Effect on Addition	162
5. Overview of Inversion Problem	165
6. Analytic Continuation	166
7. Riemann Surface of the Integrand	169
8. An Elliptic Integral	174
9. Computability of the Correspondence	183
VII. Dirichlet's Theorem	
1. Motivation	189
2. Dirichlet Series and Euler Products	192
3. Fourier Analysis on Finite Abelian Groups	199
4. Proof of Dirichlet's Theorem	201
5. Analytic Properties of Dirichlet L Functions	207
VIII. Modular Forms for $SL(2, \mathbf{Z})$	
1. Overview	221
2. Definitions and Examples	222
3. Geometry of the q Expansion	227
4. Dimensions of Spaces of Modular Forms	231
5. L Function of a Cusp Form	238
6. Petersson Inner Product	241
7. Hecke Operators	242
8. Interaction with Petersson Inner Product	250
IX. Modular Forms for Hecke Subgroups	
1. Hecke Subgroups	256
2. Modular and Cusp Forms	261
3. Examples of Modular Forms	265
4. L Function of a Cusp Form	267
5. Dimensions of Spaces of Cusp Forms	271
6. Hecke Operators	273
7. Oldforms and Newforms	283
X. L Function of an Elliptic Curve	
1. Global Minimal Weierstrass Equations	290
2. Zeta Functions and L Functions	294
3. Hasse's Theorem	296

XI. Eichler-Shimura Theory	
1. Overview	302
2. Riemann surface $X_0(N)$	311
3. Meromorphic Differentials	312
4. Properties of Compact Riemann Surfaces	316
5. Hecke Operators on Integral Homology	320
6. Modular Function $j(\tau)$	333
7. Varieties and Curves	341
8. Canonical Model of $X_0(N)$	349
9. Abstract Elliptic Curves and Isogenies	359
10. Abelian Varieties and Jacobian Variety	367
11. Elliptic Curves Constructed from $S_2(\Gamma_0(N))$	374
12. Match of L Functions	383
XII. Taniyama-Weil Conjecture	
1. Relationships among Conjectures	386
2. Strong Weil Curves and Twists	392
3. Computations of Equations of Weil Curves	394
4. Connection with Fermat's Last Theorem	397
Notes	401
References	409
Index of Notation	419
Index	423