

Contents

1. Basic Quantum Mechanics	1
1.1 Postulates of Quantum Mechanics	1
1.1.1 Postulate 1	1
1.1.2 Postulate 2	11
1.1.3 Postulate 3	11
1.1.4 Postulate 4	11
1.1.5 Postulate 5	13
1.2 Geometric Phase	16
1.2.1 Geometric Phase of a Harmonic Oscillator	18
1.2.2 Geometric Phase of a Two-Level System	18
1.2.3 Geometric Phase in Adiabatic Evolution	18
1.3 Time-Dependent Approximation Method	19
1.4 Quantum Mechanics of a Composite System	20
1.5 Quantum Mechanics of a Subsystem and Density Operator . .	21
1.6 Systems of One and Two Spin-1/2s	23
1.7 Wave-Particle Duality	26
1.8 Measurement Postulate and Paradoxes of Quantum Theory .	29
1.8.1 The Measurement Problem	30
1.8.2 Schrödinger's Cat Paradox	31
1.8.3 EPR Paradox	32
1.9 Local Hidden Variables Theory	34
2. Algebra of the Exponential Operator	37
2.1 Parametric Differentiation of the Exponential	37
2.2 Exponential of a Finite-Dimensional Operator	38
2.3 Lie Algebraic Similarity Transformations	39
2.3.1 Harmonic Oscillator Algebra	41
2.3.2 The $SU(2)$ Algebra	42
2.3.3 The $SU(1,1)$ Algebra	43
2.3.4 The $SU(m)$ Algebra	45
2.3.5 The $SU(m, n)$ Algebra	45
2.4 Disentangling an Exponential	48
2.4.1 The Harmonic Oscillator Algebra	49
2.4.2 The $SU(2)$ Algebra	50

2.4.3	<i>SU(1,1)</i> Algebra	51
2.5	Time-Ordered Exponential Integral	52
2.5.1	Harmonic Oscillator Algebra	52
2.5.2	<i>SU(2)</i> Algebra	53
2.5.3	The <i>SU(1,1)</i> Algebra	53
3.	Representations of Some Lie Algebras	55
3.1	Representation by Eigenvectors and Group Parameters	55
3.1.1	Bases Constituted by Eigenvectors.....	55
3.1.2	Bases Labeled by Group Parameters	56
3.2	Representations of Harmonic Oscillator Algebra	60
3.2.1	Orthonormal Bases	60
3.2.2	Minimum Uncertainty Coherent States	61
3.3	Representations of <i>SU(2)</i>	68
3.3.1	Orthonormal Representation	68
3.3.2	Minimum Uncertainty Coherent States	70
3.4	Representations of <i>SU(1,1)</i>	76
3.4.1	Orthonormal Bases	76
3.4.2	Minimum Uncertainty Coherent States	77
4.	Quasiprobabilities and Non-classical States	81
4.1	Phase Space Distribution Functions	81
4.2	Phase Space Representation of Spins	88
4.3	Quasiprobability Distributions for Eigenvalues of Spin Components	93
4.4	Classical and Non-classical States	95
4.4.1	Non-classical States of Electromagnetic Field	95
4.4.2	Non-classical States of Spin-1/2s	97
5.	Theory of Stochastic Processes	99
5.1	Probability Distributions	99
5.2	Markov Processes	102
5.3	Detailed Balance	105
5.4	Liouville and Fokker–Planck Equations	106
5.4.1	Liouville Equation	107
5.4.2	The Fokker–Planck Equation	107
5.5	Stochastic Differential Equations	109
5.6	Linear Equations with Additive Noise	110
5.7	Linear Equations with Multiplicative Noise	112
5.7.1	Univariate Linear Multiplicative Stochastic Differ- ential Equations	113
5.7.2	Multivariate Linear Multiplicative Stochastic Differ- ential Equations	114
5.8	The Poisson Process	115

5.9	Stochastic Differential Equation Driven by Random Telegraph Noise	116
6.	The Electromagnetic Field	119
6.1	Free Classical Field	119
6.2	Field Quantization	121
6.3	Statistical Properties of Classical Field	123
6.3.1	First-Order Correlation Function	125
6.3.2	Second-Order Correlation Function	126
6.3.3	Higher-Order Correlations	126
6.3.4	Stable and Chaotic Fields	127
6.4	Statistical Properties of Quantized Field	130
6.4.1	First-Order Correlation	131
6.4.2	Second-Order Correlation	132
6.4.3	Quantized Coherent and Thermal Fields	132
6.5	Homodyned Detection	134
6.6	Spectrum	135
7.	Atom–Field Interaction Hamiltonians	137
7.1	Dipole Interaction	137
7.2	Rotating Wave and Resonance Approximations	140
7.3	Two-Level Atom	144
7.4	Three-Level Atom	145
7.5	Effective Two-Level Atom	146
7.6	Multi-channel Models	149
7.7	Parametric Processes	150
7.8	Cavity QED	151
7.9	Moving Atom	153
8.	Quantum Theory of Damping	155
8.1	The Master Equation	155
8.2	Solving a Master Equation	160
8.3	Multi-Time Average of System Operators	162
8.4	Bath of Harmonic Oscillators	163
8.4.1	Thermal Reservoir	164
8.4.2	Squeezed Reservoir	166
8.4.3	Reservoir of the Electromagnetic Field	167
8.5	Master Equation for a Harmonic Oscillator	168
8.6	Master Equation for Two-Level Atoms	170
8.6.1	Two-Level Atom in a Monochromatic Field	171
8.6.2	Collisional Damping	172
8.7	Master Equation for a Three-Level Atom	173
8.8	Master Equation for Field Interacting with a Reservoir of Atoms	174

9. Linear and Nonlinear Response of a System in an External Field	177
9.1 Steady State of a System in an External Field	177
9.2 Optical Susceptibility	179
9.3 Rate of Absorption of Energy	181
9.4 Response in a Fluctuating Field	183
10. Solution of Linear Equations:	
Method of Eigenvector Expansion	185
10.1 Eigenvalues and Eigenvectors	186
10.2 Generalized Eigenvalues and Eigenvectors	189
10.3 Solution of Two-Term Difference-Differential Equation	191
10.4 Exactly Solvable Two- and Three-Term Recursion Relations	192
10.4.1 Two-Term Recursion Relations	192
10.4.2 Three-Term Recursion Relations	193
11. Two-Level and Three-Level Hamiltonian Systems	199
11.1 Exactly Solvable Two-Level Systems	199
11.1.1 Time-Independent Detuning and Coupling	202
11.1.2 On-Resonant Real Time-Dependent Coupling	208
11.1.3 Fluctuating Coupling	208
11.2 N Two-Level Atoms in a Quantized Field	210
11.3 Exactly Solvable Three-Level Systems	210
11.4 Effective Two-Level Approximation	212
12. Dissipative Atomic Systems	215
12.1 Two-Level Atom in a Quasimonochromatic Field	215
12.1.1 Time-Dependent Evolution Operator Reducible to $SU(2)$	217
12.1.2 Time-Independent Evolution Operator	219
12.1.3 Nonlinear Response in a Bichromatic Field	223
12.2 N Two-Level Atoms in a Monochromatic Field	224
12.3 Two-Level Atoms in a Fluctuating Field	236
12.4 Driven Three-Level Atom	237
13. Dissipative Field Dynamics	239
13.1 Down-Conversion in a Damped Cavity	239
13.1.1 Averages and Variances of the Cavity Field Operators	240
13.1.2 Density Matrix	242
13.2 Field Interacting with a Two-Photon Reservoir	245
13.2.1 Two-Photon Absorption	245
13.2.2 Two-Photon Generation and Absorption	247
13.3 Reservoir in the Lambda Configuration	248

14. Dissipative Cavity QED	251
14.1 Two-Level Atoms in a Single-Mode Cavity	251
14.2 Strong Atom-Field Coupling	252
14.2.1 Single Two-Level Atom	252
14.3 Response to an External Field	255
14.3.1 Linear Response to a Monochromatic Field	256
14.3.2 Nonlinear Response to a Bichromatic Field	257
14.4 The Micromaser	259
14.4.1 Density Operator of the Field	259
14.4.2 Two-Level Atomic Micromaser	263
14.4.3 Atomic Statistics	266
Appendices	267
A. Some Mathematical Formulae	267
B. Hypergeometric Equation	270
C. Solution of Two- and Three-Dimensional Linear Equations	272
D. Roots of a Polynomial	273
References	277
Index	283