ELASTIC SCATTERING OF ELECTRONS AND POSITRONS

In addition, the ICRU is evaluating the possibility of expanding its program to encompass nonionizing radiation, particularly the quantities and units aspects.

The Commission continually reviews radiation science with the aim of identifying areas where the development of guidance and recommendations can make an important contribution.

THE ICRU'S RELATIONSHIP WITH OTHER ORGANIZATIONS

In addition to its close relationship with the ICRP, the ICRU has developed relationships with other organizations interested in the problems of radiation quantities, units, and measurements. Since 1955, the ICRU has had an official relationship with the World Health Organization (WHO), whereby the ICRU is looked to for primary guidance in matters of radiation units and measurements and, in turn, the WHO assists in the worldwide dissemination of the Commission's recommendations. In 1960, the ICRU entered into consultative status with the International Atomic Energy Agency (IAEA). The Commission has a formal relationship with the United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR), whereby ICRU observers are invited to attend annual UNSCEAR meetings. The Commission and the International Organization for Standardization (ISO) informally exchange notifications of meetings, and the ICRU is formally designated for liaison with two of the ISO technical committees. The ICRU also enjoys a strong relationship with its sister organization, the National Council on Radiation Protection and Measurements (NCRP). In essence, these organizations were founded concurrently by the same individuals. Presently, this long-standing relationship is formally acknowledged by a special liaison agreement. The ICRU also corresponds and exchanges final reports with the following organizations:

Bureau International de Métrologie Légale Bureau International des Poids et Mesures European Commission

Council for International Organizations of Medical Sciences

Food and Agriculture Organization of the United Nations

International Committee of Photobiology
International Council of Scientific Unions
International Electrotechnical Commission
International Labor Office
International Organization for Medical Physics
International Radiation Protection Association

International Union of Pure and Applied Physics United Nations Educational, Scientific and Cultural Organization

The Commission has found its relationship with all of these organizations fruitful and of substantial benefit to the ICRU program.

OPERATING FUNDS

In recent years, principal financial support has been provided by the European Commission, the National Cancer Institute of the US Department of Health and Human Services and the International Atomic Energy Agency. In addition, during the last 10 years, financial support has been received from the following organizations:

American Association of Physicists in Medicine Belgian Nuclear Research Centre Canadian Nuclear Safety Commission Eastman Kodak Company Electricité de France Fuji Medical Systems GSF-Forschungszentrum für Umwelt un Gesundheit Hitachi, Ltd. International Radiation Protection Association International Society of Radiology Ion Beam Applications Italian Radiological Association Japan Industries Association of Radiological Systems Japanese Society of Radiological Technology MDS Nordion National Institute of Standards and Technology Nederlandse Vereniging voor Radiologie Ohio State University Philips Medical Systems, Incorporated Radiation Research Society Siemens Varian

In addition to the direct monetary support provided by these organizations, many organizations provide indirect support for the Commission's program. This support is provided in many forms, including, among others, subsidies for (1) the time of individuals participating in ICRU activities, (2) travel costs involved in ICRU meetings, and (3) meeting facilities and services.

In recognition of the fact that its work is made possible by the generous support provided by all of the organizations supporting its program, the Commission expresses its deep appreciation.

> Paul M. DeLuca Chairman, ICRU Madison, Wisconsin, USA

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PREFACE

The present Report treats cross-section data for elastic scattering of electrons and positrons traversing various materials. Those data are prerequisite for transport analyses through either Monte Carlo simulations or analytic transport theory. On issuing the Report, we cannot help first thinking of Martin J. Berger, a member of the Report Committee who died on 6 November 2004. A pioneer in studies of electron and photon transport in matter through Monte Carlo simulations, Martin contributed enormously to theoretical dosimetry and many applications of concern to the ICRU, as described in the obituary appearing in the ICRU News 2005 #1. Naturally, Martin played the leadership role in the initial conception of the Report and during its preparation by generously offering deep knowledge and sound judgment stemming from decades of experience. The high level of scholarship that we hope the reader will perceive in the following pages is largely attributable to Martin's wisdom and his influence on the Report Committee.

The theme of the Report represents an important step in the continuing endeavor of the radiation physics community toward establishing basic physical data necessary for theoretical dosimetry, by which we mean fully detailed characterization of the delivery of the energy of ionizing radiation to matter. As a leader and a major participant of this endeavor, the ICRU fostered work on basic physical data pertinent to ionizing radiation. The work so far has resulted in the following reports: Report 31, Average Energy Required to Produce an Ion Pair (1979); Report 37, Stopping Powers for Electrons and Positrons (1984); Report 46, Photon, Electron, Proton and Neutron Interaction Data for Body Tissues (1992); Report 49, Stopping Powers and Ranges for Protons and Alpha Particles (1993); Report 55, Secondary Electron Spectra from Charged Particle Interactions (1995); Report 63, Nuclear Data for Neutron and Proton Radiotherapy and for Radiation Protection (2000); and Report 73, Stopping of Ions Heavier than Helium (2005).

Stopping powers of materials for charged particles, treated in Reports 37, 49, and 73, indicate the mean energy losses from the particles per unit path length and thus are crucial in determining how deep the particles penetrate into the materials. A major fraction of the energy losses turns into the kinetic energies of secondary electrons, as fully discussed in Report 55. The differential cross-section for elastic scattering of electrons by atoms and molecules in the materials describes how much the direction of electron motion changes upon each elastic-scattering event. Consequently, it governs the spatial characteristics of electron tracks, and hence of the energy delivery to matter. Here lies the significance of the extensive numerical data presented in the present Report.

In effect, the reports cited above form a series. By its nature, the general theme of this series of reports must be continuously pursued, to make certain that our recommendations remain scientifically valid, internationally acceptable, and therefore meaningful. In other words, the topics of this series of reports must be revisited and updated from time to time in the light of new scientific knowledge. With this idea in mind, the ICRU is beginning a new program for reviewing key data, that is, carefully selected kinds of data concerning selected materials that are most important as theoretical input in the realization of measurement standards for ionizing radiation.

Finally, we are sad to note the death of Lewis V. Spencer on 11 November 2005, who was a pioneer in theoretical dosimetry and in electron-transport theory and was awarded the Gray Medal by the ICRU in 1969. The ICRU dedicates the present volume to the memory of Lew, as an expression of the deepest respect for his scientific achievements and of the warmest affection to him.

André Wambersie Mitio Inokuti Stephen M. Seltzer

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