Contents

1 Introduction 1
References 4

2 Spontaneous Breaking of Space Translation Symmetry 7
2.1 Spontaneous Formation of Space Crystals 7
2.1.1 Consequences of Continuous Space Translation Symmetry .. 7
2.1.2 Spontaneous Emergence of Crystals in Space 9
2.2 Spontaneous Breaking of Space Translation Symmetry in a Non-interacting System 14
References 16

3 Spontaneous Breaking of Continuous Time Translation Symmetry ... 19
3.1 Time Crystal Idea 19
3.1.1 Wilczek Model 21
3.1.2 Excited Eigenstates and Time Crystals Formation 25
3.1.3 Lack of Time Crystal Behavior of Systems with Two-Body Interactions in the Ground State and in the Equilibrium State 27
3.1.4 Time Crystal Behavior of Systems with Multi-Body Interactions 28
3.1.5 Classical Time Crystals 31
3.2 Josephson Effect and Superfluid Phase Oscillations 33
3.2.1 Josephson Effect 33
3.2.2 Superfluid Phase Oscillations 35
References 36

4 Discrete Time Crystals and Related Phenomena 39
4.1 Floquet Formalism and Discrete Time Translation Symmetry 39
4.1.1 Floquet Formalism 39
4.1.2 Discrete Time Translation Symmetry of Periodically Driven Systems 46

4.2 Discrete Time Crystals in Ultra-Cold Atoms Bouncing on an Oscillating Atom Mirror 46
4.2.1 A Particle Bouncing Resonantly on an Oscillating Mirror: Classical and Quantum Description 47
4.2.2 Formation of Discrete Time Crystals 56
4.2.3 Phase Diagram 63
4.2.4 Dramatic Breaking of the Discrete Time Translation Symmetry and Formation of n-Tupling Discrete Time Crystals 72
4.2.5 Stability of Discrete Time Crystals in Ultra-Cold Atoms Bouncing on an Oscillating Mirror 77
4.3 Discrete Time Crystals in Periodically Driven Lattice Systems 81
4.3.1 Discrete Time Crystals in Spin Systems with Disorder 81
4.3.2 Clean Discrete Time Crystals and Prethermal Character of Discrete Time Crystals in Lattice Systems 87
4.4 Fractional Time Crystals 99
4.4.1 Fractional Time Crystals in Ultra-Cold Atoms Bouncing on a Mirror 100
4.4.2 Fractional Time Crystal in Spin Systems 104
4.5 Discrete Time Quasi-Crystals 108
4.6 Experimental Realizations 115
4.6.1 Periodically Driven Chain of Ions 116
4.6.2 Periodically Driven Nitrogen-Vacancy Centers in Diamond . 119
4.6.3 Periodically Driven Nuclear Magnetic Moments 123
4.6.4 Space-Time Crystals in an Atomic Bose–Einstein Condensate 127
4.7 Dissipative Systems 133
4.7.1 Dicke Time Crystals in Driven Dissipative Systems 134
4.7.2 Discrete Time Crystals in Dissipative Systems in the Absence of Manifest Symmetries 140
4.7.3 Analysis of Discrete Time Crystals in Open Systems 147
4.7.4 Breaking of Continuous Time Translation Symmetry in Dissipative Systems 151
4.7.5 Imaginary Time Crystals 155
4.8 Further Reading 161
References 162

5 Condensed Matter Physics in the Time Dimension 173
5.1 Platform for Condensed Matter Physics in the Time Dimension 173
5.2 Anderson Localization in the Time Dimension 179
5.2.1 Anderson Localization in Time Crystals 181
5.2.2 Anderson Localization in the Time Dimension Without Crystalline Structure in Time 187
5.2.3 Anderson Localized-Delocalized Transition in the Time Dimension 192

5.3 Single-Particle Condensed Matter Phenomena in the Time Dimension 200
5.3.1 Topological Time Crystals 200
5.3.2 Engineering of Effective Potentials: Quasi-Crystalline Structure in the Time Dimension 210
5.4 Many-Body Condensed Matter Physics in the Time Dimension 212
5.4.1 Mott Insulator Phase in the Time Dimension 212
5.4.2 Many-Body Localization with Temporal Disorder 216
5.4.3 Engineering of Long-Range Interactions in Time Crystals ... 219
5.4.4 Time Lattices with Properties of Multi-Dimensional Space Crystals 223
5.5 Time Engineering: Anderson Molecules 226
References 229

6 Phase Space Crystals 237
6.1 Idea of Phase Space Crystals 237
6.2 Topological Phase Space Crystals 240
6.3 Many-Body Physics in Phase Space Crystals 245
References 248

7 Photonic Time Crystals 251
7.1 Photonic Crystals in Space 251
7.1.1 Introduction to Photonic Crystals in Space 251
7.1.2 Topological Properties of Photonic Crystals in Space 254
7.2 Photonic Crystals in Time 258
7.2.1 Introduction to Photonic Time Crystals 259
7.2.2 Topological Properties of Photonic Time Crystals 262
References 264

Index 267