

Contents

Preface	v
Preface to the second edition	vii
Part I	
NEWTONIAN MECHANICS	1
Chapter 1	
Experimental facts	3
1. The principles of relativity and determinacy	3
2. The galilean group and Newton's equations	4
3. Examples of mechanical systems	11
Chapter 2	
Investigation of the equations of motion	15
4. Systems with one degree of freedom	15
5. Systems with two degrees of freedom	22
6. Conservative force fields	28
7. Angular momentum	30
8. Investigation of motion in a central field	33
9. The motion of a point in three-space	42
10. Motions of a system of n points	44
11. The method of similarity	50
Part II	
LAGRANGIAN MECHANICS	53
Chapter 3	
Variational principles	55
12. Calculus of variations	55
13. Lagrange's equations	59

14. Legendre transformations	61
15. Hamilton's equations	65
16. Liouville's theorem	68

Chapter 4

Lagrangian mechanics on manifolds	75
--	-----------

17. Holonomic constraints	75
18. Differentiable manifolds	77
19. Lagrangian dynamical systems	83
20. E. Noether's theorem	88
21. D'Alembert's principle	91

Chapter 5

Oscillations	98
---------------------	-----------

22. Linearization	98
23. Small oscillations	103
24. Behavior of characteristic frequencies	110
25. Parametric resonance	113

Chapter 6

Rigid bodies	123
---------------------	------------

26. Motion in a moving coordinate system	123
27. Inertial forces and the Coriolis force	129
28. Rigid bodies	133
29. Euler's equations. Poincaré's description of the motion	142
30. Lagrange's top	148
31. Sleeping tops and fast tops	154

Part III

HAMILTONIAN MECHANICS	161
------------------------------	------------

Chapter 7

Differential forms	163
---------------------------	------------

32. Exterior forms	163
33. Exterior multiplication	170
34. Differential forms	174
35. Integration of differential forms	181
36. Exterior differentiation	188

Chapter 8

Symplectic manifolds	201
-----------------------------	------------

37. Symplectic structures on manifolds	201
38. Hamiltonian phase flows and their integral invariants	204
39. The Lie algebra of vector fields	208
40. The Lie algebra of hamiltonian functions	214

41. Symplectic geometry	219
42. Parametric resonance in systems with many degrees of freedom	225
43. A symplectic atlas	229

Chapter 9

Canonical formalism	233
----------------------------	------------

44. The integral invariant of Poincaré–Cartan	233
45. Applications of the integral invariant of Poincaré–Cartan	240
46. Huygens' principle	248
47. The Hamilton–Jacobi method for integrating Hamilton's canonical equations	258
48. Generating functions	266

Chapter 10

Introduction to perturbation theory	271
--	------------

49. Integrable systems	271
50. Action-angle variables	279
51. Averaging	285
52. Averaging of perturbations	291

Appendix 1

Riemannian curvature	301
-----------------------------	------------

Appendix 2

Geodesics of left-invariant metrics on Lie groups and the hydrodynamics of ideal fluids	318
--	------------

Appendix 3

Symplectic structures on algebraic manifolds	343
---	------------

Appendix 4

Contact structures	349
---------------------------	------------

Appendix 5

Dynamical systems with symmetries	371
--	------------

Appendix 6

Normal forms of quadratic hamiltonians	381
---	------------

Appendix 7

Normal forms of hamiltonian systems near stationary points and closed trajectories	385
---	------------

Appendix 8

Theory of perturbations of conditionally periodic motion, and Kolmogorov's theorem	399
---	------------

Contents

Appendix 9

Poincaré's geometric theorem, its generalizations and applications	416
--	-----

Appendix 10

Multiplicities of characteristic frequencies, and ellipsoids depending on parameters	425
--	-----

Appendix 11

Short wave asymptotics	438
------------------------	-----

Appendix 12

Lagrangian singularities	446
--------------------------	-----

Appendix 13

The Korteweg–de Vries equation	453
--------------------------------	-----

Appendix 14

Poisson structures	456
--------------------	-----

Appendix 15

On elliptic coordinates	469
-------------------------	-----

Appendix 16

Singularities of ray systems	480
------------------------------	-----

Index	511
-------	-----