Contents

1 Forecasting and Chaos 1
1.1 Historical Introduction 1
1.1.1 The Scientific Method 1
1.1.2 Forecasting and Determinism 3
1.1.3 Human Knowledge and Decision-Making 9
1.2 Chaotic Dynamics 10
1.3 Computer Numerical Explorations 14
1.3.1 Solving ODEs Numerically 15
1.3.2 Solving PDEs Numerically 16
1.3.3 Numerical Forecast 17
1.3.4 Reliability and Stability of Numerical Schemes 20
1.3.5 Stiffness 22
1.3.6 Symplectic Integrators 23
1.4 Shadowing and Predictability 25
1.5 Concluding Remarks 28
References 28

2 Lyapunov Exponents 33
2.1 Lyapunov Exponents 33
2.2 The Lyapunov Spectrum 36
2.3 The Lyapunov Exponents Family 38
2.4 Local and Non-local Timescales: Covariant Vectors 42
2.5 Finite-Time Exponents 44
2.6 Distributions of Finite-Time Exponents 45
2.7 The Harmonic Oscillator 47
2.8 The Rössler System 48
2.9 The Hénon-Heiles System 55
2.10 Concluding Remarks 66
References 66

3 Dynamical Regimes and Timescales 71
3.1 Temporal Evolution 71
3.2 Regime Identification 72
3.3 Transient Behaviours, Sticky Orbits and Transient Chaos 74
3.4 The Hénon-Heiles System 75
3.5 The Contopoulos System 78
3.6 The Rössler System 90
3.7 Hyperbolicity Characterisation Through Finite-Time Exponents ... 92
3.8 Concluding Remarks 96
References 97

4 Predictability 101
4.1 Numerical Predictability 101
4.1.1 Predictability, Attractors and Basins 101
4.1.2 Predictability and Trajectories 104
4.2 The Predictability Index 107
4.2.1 The Hénon-Heiles System 109
4.2.2 The Contopoulos System 117
4.2.3 The Rössler System 120
4.3 Concluding Remarks 125
References 128

5 Chaos, Predictability and Astronomy 131
5.1 Introduction 131
5.2 Numerical Forecasting in Astronomy 131
5.3 Time Series Analyses 135
5.4 Celestial Mechanics 137
5.5 Sitnikov Problem 138
5.6 Predictability and Stability in the Solar System 140
5.7 Stellar Systems 144
5.8 Concluding Remarks 148
References 148

6 A Detailed Example: Galactic Dynamics 151
6.1 Introduction 151
6.2 Chaos in Galactic Astronomy 152
6.3 Predictability in a Galactic System 153
6.4 Role of Dark Matter Haloes in Predictability 163
6.4.1 Galactic Orbits 164
6.4.2 Role of the Dark Halo Orientation 165
6.4.3 Role of the Dark Halo Flattening 174
6.5 Analysis of the Results 179
6.5.1 Dark Halo Orientation 181
6.5.2 Flattening 182
6.6 Concluding Remarks 183
References 186

A Numerical Calculation of Lyapunov Exponents 189
A.1 The Variational Equation 189
A.2 Selection of Initial Perturbations 192
A.3 Other Methods 194
A.4 Practical Implementation for Building the Finite-Time Distributions 195
References 195