Contents

PREFACE TO VOLUME III xvi

NOTATION xx

24 HISTORICAL INTRODUCTION 1

24.1 Unconventional Symmetries and 'No-Go' Theorems 1

$SU(6)$ symmetry □ Elementary no-go theorem for unconventional semi-simple compact Lie algebras □ Role of relativity

24.2 The Birth of Supersymmetry 4

Bosonic string theory □ Fermionic coordinates □ Worldsheet supersymmetry □ Wess–Zumino model □ Precursors

Appendix A $SU(6)$ Symmetry of Non-Relativistic Quark Models 8

Appendix B The Coleman–Mandula Theorem 12

Problems 22

References 22

25 SUPERSYMMETRY ALGEBRAS 25

25.1 Graded Lie Algebras and Graded Parameters 25

Fermionic and bosonic generators □ Super-Jacobi identity □ Grassmann parameters □ Structure constants from supergroup multiplication rules □ Complex conjugates

25.2 Supersymmetry Algebras 29

Haag–Lopuszanski–Sohnius theorem □ Lorentz transformation of fermionic generators □ Central charges □ Other bosonic symmetries □ R-symmetry □ Simple

and extended supersymmetry □ Four-component notation □ Superconformal algebra

25.3 Space Inversion Properties of Supersymmetry Generators 40

Parity phases in simple supersymmetry □ Fermions have imaginary parity □ Parity matrices in extended supersymmetry □ Dirac notation

25.4 Massless Particle Supermultiplets 43

Known particles are massless for unbroken supersymmetry □ Helicity raising and lowering operators □ Simple supersymmetry doublets □ Squarks, sleptons, and gauginos □ Gravitino □ Extended supersymmetry multiplets □ Chirality problem for extended supersymmetry

25.5 Massive Particle Supermultiplets 48

Raising and lowering operators for spin 3-component □ General massive multiplets for simple supersymmetry □ Collapsed supermultiplet □ Mass bounds in extended supersymmetry □ BPS states and short supermultiplets

Problems 53

References 54

26 SUPERSYMMETRIC FIELD THEORIES 55

26.1 Direct Construction of Field Supermultiplets 55

Construction of simplest $N = 1$ field multiplet □ Auxiliary field □ Infinitesimal supersymmetry transformation rules □ Four-component notation □ Wess–Zumino supermultiplets regained

26.2 General Superfields 59

Superspace spinor coordinates □ Supersymmetry generators as superspace differential operators □ Supersymmetry transformations in superspace □ General superfields □ Multiplication rules □ Supersymmetric differential operators in superspace □ Supersymmetric actions for general superfields □ Parity of component fields □ Counting fermionic and bosonic components

26.3 Chiral and Linear Superfields 68

Chirality conditions on a general superfield □ Left- and right-chiral superfields □ Coordinates $x_{\pm}^{\mu}$ □ Differential constraints □ Product rules □ Supersymmetric $\mathscr{F}$-terms □ $\mathscr{F}$-terms equivalent to D-terms □ Superpotentials □ Kahler potentials □ Partial integration in superspace □ Space inversion of chiral superfields □ R-symmetry again □ Linear superfields

26.4 Renormalizable Theories of Chiral Superfields 75

Counting powers □ Kinematic Lagrangian □ $\mathscr{F}$-term of the superpotential □ Complete Lagrangian □ Elimination of auxiliary fields □ On-shell superalgebra □ Vacuum solutions □ Masses and couplings □ Wess–Zumino Lagrangian regained

26.5 Spontaneous Supersymmetry Breaking in the Tree Approximation 83

O'Raifeartaigh mechanism □ R-symmetry constraints □ Flat directions □ Goldstino

26.6 Superspace Integrals, Field Equations, and the Current Superfield 86

Berezin integration □ D- and $\mathscr{F}$-terms as superspace integrals □ Potential superfields □ Superspace field equations □ Conserved currents as components of linear superfields □ Conservation conditions in superspace

26.7 The Supercurrent 90

Supersymmetry current □ Superspace transformations generated by the supersymmetry current □ Local supersymmetry transformations □ Construction of the supercurrent □ Conservation of the supercurrent □ Energy-momentum tensor and R-current □ Scale invariance and R conservation □ Non-uniqueness of supercurrent

26.8 General Kahler Potentials* 102

Non-renormalizable non-derivative actions □ D-term of Kahler potential □ Kahler metric □ Lagrangian density □ Non-linear σ-models from spontaneous internal symmetry breaking □ Kahler manifolds □ Complexified coset spaces

Appendix Majorana Spinors 107

Problems 111

References 112

27 SUPERSYMMETRIC GAUGE THEORIES 113

27.1 Gauge-Invariant Actions for Chiral Superfields 113

Gauge transformation of chiral superfields □ Gauge superfield V □ Extended gauge invariance □ Wess–Zumino gauge □ Supersymmetric gauge-invariant kinematic terms for chiral superfields

27.2 Gauge-Invariant Action for Abelian Gauge Superfields 122

Field strength supermultiplet □ Kinematic Lagrangian density for Abelian gauge supermultiplet □ Fayet–Iliopoulos terms □ Abelian field-strength spinor superfield W_{α} □ Left- and right-chiral parts of W_{α} □ W_{α} as a superspace derivative of V □ Gauge invariance of W_{α} □ 'Bianchi' identities in superspace

27.3 Gauge-Invariant Action for General Gauge Superfields 127

Kinematic Lagrangian density for non-Abelian gauge supermultiplet □ Non-Abelian field-strength spinor superfield $W_{A\alpha}$ □ Left- and right-chiral parts of $W_{A\alpha}$ □ θ-term □ Complex coupling parameter τ

27.4 Renormalizable Gauge Theories with Chiral Superfields 132

Supersymmetric Lagrangian density □ Elimination of auxiliary fields □ Conditions for unbroken supersymmetry □ Counting independent conditions and field

variables □ Unitarity gauge □ Masses for spins 0, 1/2, and 1 □ Supersymmetry current □ Non-Abelian gauge theories with general Kahler potentials □ Gaugino mass

27.5 Supersymmetry Breaking in the Tree Approximation Resumed 144

Supersymmetry breaking in supersymmetric quantum electrodynamics □ General case: masses for spins 0, 1/2, and 1 □ Mass sum rule □ Goldstino component of gaugino and chiral fermion fields

27.6 Perturbative Non-Renormalization Theorems 148

Non-renormalization of Wilsonian superpotential □ One-loop renormalization of terms quadratic in gauge superfields □ Proof using holomorphy and new symmetries with external superfields □ Non-renormalization of Fayet–Iliopoulos constants ξ_A □ For $\xi_A = 0$, supersymmetry breaking depends only on superpotential □ Non-renormalizable theories

27.7 Soft Supersymmetry Breaking* 155

Limitation on supersymmetry-breaking radiative corrections □ Quadratic divergences in tadpole graphs

27.8 Another Approach: Gauge-Invariant Supersymmetry Transformations 157

De Wit–Freedman transformation rules □ Preserving Wess–Zumino gauge with combined supersymmetry and extended gauge transformations

27.9 Gauge Theories with Extended Supersymmetry* 160

$N = 2$ supersymmetry from $N = 1$ supersymmetry and R-symmetry □ Lagrangian for $N = 2$ supersymmetric gauge theory □ Eliminating auxiliary fields □ Supersymmetry currents □ Witten–Olive calculation of central charge □ Non-renormalization of masses □ BPS monopoles □ Adding hypermultiplets □ $N = 4$ supersymmetry □ Calculation of beta function □ $N = 4$ theory is finite □ Montonen–Olive duality

Problems 175

References 176

28 SUPERSYMMETRIC VERSIONS OF THE STANDARD MODEL 179

28.1 Superfields, Anomalies, and Conservation Laws 180

Quark, lepton, and gauge superfields □ At least two scalar doublet superfields □ $\mathscr{F}$-term Yukawa couplings □ Constraints from anomalies □ Unsuppressed violation of baryon and lepton numbers □ R-symmetry □ R parity □ μ-term □ Hierarchy problem □ Sparticle masses □ Cosmological constraints on lightest superparticle

28.2 Supersymmetry and Strong–Electroweak Unification 188

Renormalization group equations for running gauge couplings □ Effect of supersymmetry on beta functions □ Calculation of weak mixing angle and unification mass □ Just two scalar doublet superfields □ Coupling at unification scale

28.3 Where is Supersymmetry Broken? 192

Tree approximation supersymmetry breakdown ruled out □ Hierarchy from non-perturbative effects of asymptotically free gauge couplings □ Gauge and gravitational mediation of supersymmetry breaking □ Estimates of supersymmetry-breaking scale □ Gravitino mass □ Cosmological constraints

28.4 The Minimal Supersymmetric Standard Model 198

Supersymmetry breaking by superrenormalizable terms □ General Lagrangian □ Flavor changing processes □ Calculation of $K^0 \leftrightarrow \overline{K}^0$ □ Degenerate squarks and sleptons □ CP violation □ Calculation of quark chromoelectric dipole moment □ 'Naive dimensional analysis' □ Neutron electric dipole moment □ Constraints on masses and/or phases

28.5 The Sector of Zero Baryon and Lepton Number 209

D-term contribution to scalar potential □ μ-term contribution to scalar potential □ Soft supersymmetry breaking terms □ Vacuum stability constraint on parameters □ Finding a minimum of potential □ $B\mu \neq 0$ □ Masses of CP-odd neutral scalars □ Masses of CP-even neutral scalars □ Masses of charged scalars □ Bounds on masses □ Radiative corrections □ Conditions for electroweak symmetry breaking □ Charginos and neutralinos □ Lower bound on $|\mu|$

28.6 Gauge Mediation of Supersymmetry Breaking 220

Messenger superfields □ Supersymmetry breaking in gauge supermultiplet propagators □ Gaugino masses □ Squark and slepton masses □ Derivation from holomorphy □ Radiative corrections □ Numerical examples □ Higgs scalar masses □ μ problem □ A_{ij} and C_{ij} parameters □ Gravitino as lightest sparticle □ Next-to-lightest sparticle

28.7 Baryon and Lepton Non-Conservation 235

Dimensionality five interactions □ Gaugino exchange □ Gluino exchange suppressed □ Wino and bino exchange effects □ Estimate of proton lifetime □ Favored modes of proton decay

Problems 240

References 241

29 BEYOND PERTURBATION THEORY 248

29.1 General Aspects of Supersymmetry Breaking 248

Finite volume □ Vacuum energy and supersymmetry breaking □ Partially broken extended supersymmetry? □ Pairing of bosonic and fermionic states □ Pairing of vacuum and one-goldstino state □ Witten index □ Supersymmetry unbroken

in the Wess–Zumino model □ Models with unbroken supersymmetry and zero Witten index □ Large field values □ Weighted Witten indices

29.2 Supersymmetry Current Sum Rules 256

Sum rule for vacuum energy density □ One-goldstino contribution □ The supersymmetry-breaking parameter F □ Soft goldstino amplitudes □ Sum rule for supersymmetry current-fermion spectral functions □ One-goldstino contribution □ Vacuum energy density in terms of $\mathscr{F}$ and D vacuum values □ Vacuum energy sum rule for infinite volume

29.3 Non-Perturbative Corrections to the Superpotential 266

Non-perturbative effects break external field translation and R-conservation □ Remaining symmetry □ Example: generalized supersymmetric quantum chromodynamics □ Structure of induced superpotential for $C_1 > C_2$ □ Stabilizing the vacuum with a bare superpotential □ Vacuum moduli in generalized supersymmetric quantum chromodynamics for $N_c > N_f$ □ Induced superpotential is linear in bare superpotential parameters for $C_1 = C_2$ □ One-loop renormalization of $[W_\alpha W_\alpha]_\mathscr{F}$ term for all C_1, C_2

29.4 Supersymmetry Breaking in Gauge Theories 276

Witten index vanishes in supersymmetric quantum electrodynamics □ C-weighted Witten index □ Supersymmetry unbroken in supersymmetric quantum electrodynamics □ Counting zero-energy gauge field states in supersymmetric quantum electrodynamics □ Calculating Witten index for general supersymmetric pure gauge theories □ Counting zero-energy gauge field states for general supersymmetric pure gauge theories □ Weyl invariance □ Supersymmetry unbroken in general supersymmetric pure gauge theories □ Witten index and R anomalies □ Adding chiral scalars □ Model with spontaneously broken supersymmetry

29.5 The Seiberg–Witten Solution* 287

Underlying $N = 2$ supersymmetric Lagrangian □ Vacuum modulus □ Leading non-renormalizable terms in the effective Lagrangian □ Effective Lagrangian for component fields □ Kahler potential and gauge coupling from a function $h(\Phi)$ □ $SU(2)$ R-symmetry □ Prepotential □ Duality transformation □ $h(\Phi)$ translation □ $\mathbb{Z}_8$ R-symmetry □ $SL(2,\mathbb{Z})$-symmetry □ Central charge □ Charge and magnetic monopole moments □ Perturbative behavior for large $|a|$ □ Monodromy at infinity □ Singularities from dyons □ Monodromy at singularities □ Seiberg–Witten solution □ Uniqueness proof

Problems 305

References 305

30 SUPERGRAPHS 307

30.1 Potential Superfields 308

Problem of chiral constraints □ Corresponding problem in quantum electrodynamics □ Path integrals over potential superfields

30.2 Superpropagators 310

A troublesome invariance □ Change of variables □ Defining property of superpropagator □ Analogy with quantum electrodynamics □ Propagator for potential superfields □ Propagator for chiral superfields

30.3 Calculations with Supergraphs 313

Superspace quantum effective action □ Locality in fermionic coordinates □ D-terms and $\mathscr{F}$-terms in effective action □ Counting superspace derivatives □ No renormalization of $\mathscr{F}$-terms

Problems 316

References 316

31 SUPERGRAVITY 318

31.1 The Metric Superfield 319

Vierbein formalism □ Transformation of gravitational field □ Transformation of gravitino field □ Generalized transformation of metric superfield H_μ □ Interaction of H_μ with supercurrent □ Invariance of interaction □ Generalized transformation of H_μ components □ Auxiliary fields □ Counting components □ Interaction of H_μ component fields □ Normalization of action

31.2 The Gravitational Action 326

Einstein superfield E_μ □ Component fields of E_μ □ Lagrangian for H_μ □ Value of κ □ Total Lagrangian □ Vacuum energy density □ Minimum vacuum energy □ De Sitter and anti-de Sitter spaces □ Why vacuum energy is negative □ Stability of flat space □ Weyl transformation

31.3 The Gravitino 333

Irreducibility conditions on gravitino field □ Gravitino propagator □ Gravitino kinematic Lagrangian □ Gravitino field equation □ Gravitino mass from broken supersymmetry □ Gravitino mass from s and p

31.4 Anomaly-Mediated Supersymmetry Breaking 337

First-order interaction with scale non-invariance superfield X □ General formula for X □ General first-order interaction □ Gaugino masses □ Gluino mass □ B parameter □ Wino and bino masses □ A parameters

31.5 Local Supersymmetry Transformations 341

Wess–Zumino gauge for metric superfield □ Local supersymmetry transformations □ Invariance of action

31.6 Supergravity to All Orders 343

Local supersymmetry transformation of vierbein, gravitino, and auxiliary fields □ Extended spin connection □ Local supersymmetry transformation of general scalar supermultiplet □ Product rules for general superfields □ Real matter superfields □ Chiral matter superfields □ Product rules for chiral superfields □ Cosmological constant and gravitino mass □ Lagrangian for supergravity and chiral fields with general Kahler potential and superpotential □ Elimination of auxiliary fields □ Kahler metric □ Weyl transformation □ Scalar field potential □ Conditions for flat space and unbroken supersymmetry □ Complete bosonic Lagrangian □ Canonical normalization □ Combining superpotential and Kahler potential □ No-scale models

31.7 Gravity-Mediated Supersymmetry Breaking 355

Early theories with hidden sectors □ Hidden sector gauge coupling strong at energy Λ □ First version: Observable and hidden sectors □ Separable bare superpotential □ General potential □ Terms of order $\kappa^4\Lambda^8 \approx m_g^4$ □ Λ estimated as $\approx 10^{11}$ GeV □ μ- and $B\mu$-terms □ Squark and slepton masses □ Gaugino masses □ A-parameters □ Second version: Observable, hidden, and modular sectors □ Dynamically induced superpotential for modular superfields □ Effective superpotential of observable sector □ μ-term □ Potential of observable sector scalars □ Terms of order $\kappa^8\Lambda^{12} \approx m_g^4$ □ Soft supersymmetry-breaking terms □ Λ estimated as $\approx 10^{13}$ GeV □ Shifts in modular fields □ Absence of C_{ij} terms □ Squark and slepton masses □ Gaugino masses

Appendix The Vierbein Formalism 375

Problems 378

References 379

32 SUPERSYMMETRY ALGEBRAS IN HIGHER DIMENSIONS 382

32.1 General Supersymmetry Algebras 382

Classification of fermionic generators □ Definition of weight □ Fermionic generators in fundamental spinor representation □ Fermionic generators commute with P_μ □ General form of anticommutation relations □ Central charges □ Anticommutation relations for odd dimensionality □ Anticommutation relations for even dimensionality □ R-symmetry groups

32.2 Massless Multiplets 393

Little group $O(d-2)$ □ Definition of 'spin' j □ Exclusion of $j > 2$ □ Missing fermionic generators □ Number of fermionic generators ≤ 32 □ $N = 1$ supersymmetry for $d = 11$ □ Three-form massless particle □ Types IIA, IIB and heterotic supersymmetry for $d = 10$

32.3 p-Branes 397

New conserved tensors □ Fermionic generators still in fundamental spinor representation □ Fermionic generators still commute with P_μ □ Symmetry conditions on tensor central charges □ 2-form and 5-form central charges for $d = 11$

Appendix Spinors in Higher Dimensions 401

Problems 407

References 407

AUTHOR INDEX 411

SUBJECT INDEX 416