Contents

•

1	Intro	luction		
	1.1	General	Remarks	
	1.2	Overvie	w	
		1.2.1	On Mathematics. Evolution and Projection	
		1.2.2	Electromagnetic Waveguides	
		1.2.3	Solitonics	
		1.2.4	Hydrodynamics	
		1.2.5	Guide Propagation and Interaction of Plasma	
			Waves. Metamaterials	
		1.2.6	Nanowaveguides. Bloch Waves	
		1.2.7	Microwaveguides Versus Nanowaveguides.	
			Domain Wall Propagation	
		1.2.8	Kinetics of Charges in Waveguides. Charge	
			Transport	
	Refere	ences		
2	Evolu	tion Ope	erator and Projectors to Its Eigenspaces	
	2.1	Homoge	neous Background	
		2.1.1	Formulation of the Linear Cauchy Problem	
		2.1.2	Transition to the k-Domain	
		2.1.3	Solution in the <i>k</i> -Domain. Projection Operators	
		2.1.4	Inhomogeneous Equation and Perturbations	
		2.1.5	Formulating the Nonlinear Cauchy Problem.	
			Mode Interaction	
	2.2	Boundar	y Regime Propagation in a 1D Medium	
		2.2.1	Boundary Regime Propagation in a 1D Homogeneous	
			Medium	
		2.2.2	Boundary Regime Propagation in a 1D Medium	
			with Exponential Stratification	
	2.3	General	3D Waveguide	

Contents

	2.4	Accour	nting for Nonlinear Terms	27
		2.4.1	1+1 Dimensions	27
		2.4.2	1+3 Dimensions	- 30
	2.5	Weakly	/ Inhomogeneous Propagation Medium	- 30
		2.5.1	Formulation of the Problem	- 30
		2.5.2	Expansions and Approximations	32
		2.5.3	1D Acoustic Waves	34
	Refer	ences		35
3	Elect	romagne	etic Waveguides	37
	3.1	Planar	Wayeguides	37
		3.1.1	Dielectric Slab as a Wayeguide	37
		312	Novel Experiments in Dielectric Guides	40
	32	Directe	d Modes in Rectangular Wayeguides Polarization	10
	5.2	Dispers	sion and Nonlinearity	42
		321	Maxwell's Equations for Matter Inside	12
		5.2.1	a Waveguide	42
		322	Boundary Conditions Evolution of Transverse	12
		5.2.2	Waveguide Modes	43
		323	Formulating the Boundary Regime Propagation	
		5.2.5	Problem for the Transverse Modes	46
		321	Projection Operators	51
		3.2.4	Taking into Account Weak Nonlinearity	54
	22	Cylinds	ricel Dielectric Weyequides	57
	5.5	2 2 1	Solution of the Linear Problem and the Way	57
		5.5.1	to the Projection Procedure	57
		222	Ways Train Evolution Disparsion and Interaction	- 57
		3.3.2	Wave Train Evolution, Dispersion, and Interaction	01
		3.3.3		04
	D	5.5.4		08
	Refer	ences		13
4	Wave	eguide M	10de Interactions. Coupled Nonlinear Schrödinger	
	Equa	tions		75
	4.1	Couple	d Nonlinear Schrödinger Equation	75
		4.1.1	Outline	75
		4.1.2	Conservation Laws	76
		4.1.3	Discretization of the CNS System. Explicit	
			Scheme	78
		4.1.4	Implicit Scheme	79
	4.2	Stabilit	у	79
	4.3	Conver	gence	82
	4.4	Numeri	ical Results	84
		4.4.1	Nonlinear Schrödinger Equation	84
		4.4.2	Manakov Solitons	85

		4.4.3	Collision of Two Solitons	85
		4.4.4	Different Group Velocities	87
		4.4.5	Difference Between Explicit and Implicit Schemes	88
		4.4.6	Asymptotic Solution	89
	Refer	ences		91
5	Solitonics			
	5.1	Nonlin	ear Resonance: <i>N</i> -wave Interactions	93
		5.1.1	DT Integration and the Example of Inclined	
			Solitons	93
		5.1.2	Twofold DT Application	96
	5.2	Linear	Resonance: Maxwell–Bloch Equations	100
		5.2.1	Derivation and Rescaling	100
		5.2.2	Integrability and Solutions of MB Equations	102
		5.2.3	Solutions Over Nonzero Backgrounds	105
		5.2.4	Periodic-Seed Solutions	106
		5.2.5	Integrability of the Nonreduced MB Equation	108
	5.3	Nonres	sonant Propagation (Kerr Effect): Manakov Solitons	111
		5.3.1	Lax Pair and Periodic-Seed Solutions	111
		5.3.2	Determinant Representation	113
		5.3.3	Perturbations of the Solutions	115
	Refer	ences		116
		enees		110
6	Wave	eguide P	Propagation in Hydrodynamics	119
6	Wave 6.1	eguide P Fundar	Propagation in Hydrodynamics	119 119
6	Wave 6.1	eguide P Fundar 6.1.1	Propagation in Hydrodynamics	119 119 119 122
6	Wave 6.1	eguide P Fundar 6.1.1 6.1.2	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves	119 119 122 123
6	Wave 6.1	eguide P Fundar 6.1.1 6.1.2 6.1.3	Propagation in Hydrodynamics	119 119 122 123 123
6	Wave 6.1	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves lized Fluid Dynamics	119 119 122 123 123 124
6	Wave 6.1	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves lized Fluid Dynamics Introductory Notes	119 119 122 123 123 124 124
6	Wave 6.1 6.2	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves ulized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model	119 119 122 123 123 124 124 124
6	Wave 6.1	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Ilized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function.	119 119 122 123 123 124 124 124
6	Wave 6.1	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves lized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System	119 119 122 123 123 124 124 124 126
6	Wave 6.1 6.2 6.3	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Lized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System	119 119 122 123 123 124 124 124 126 127
6	Wave 6.1 6.2 6.3	eguide P Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru Using	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Lized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method	119 119 122 123 123 124 124 126 127
6	Wave 6.1 6.2 6.3 6.4	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru Using	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Long Atmospheric Internal Waves Ilized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves	119 119 122 123 123 124 124 124 126 127 129 133
6	Wave 6.1 6.2 6.3 6.4 6.5	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru- Using Oceani Couple	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Ilized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves ed KdV Equations for a Laboratory Experiment	119 119 122 123 123 124 124 124 126 127 129 133 138
6	Wave 6.1 6.2 6.3 6.4 6.5	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru- Using Oceani Couple 6.5.1	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Ilized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves ed KdV Equations for a Laboratory Experiment Physical Model of the McEwan Experiment	119 119 122 123 123 124 124 124 126 127 129 133 138 138
6	Wave 6.1 6.2 6.3 6.4 6.5	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru- Using Oceani Couple 6.5.1 6.5.2	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves ulized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves ed KdV Equations for a Laboratory Experiment. Solution of the Sturm–Liouville Problem. Initial	119 119 122 123 123 124 124 124 126 127 129 133 138 138
6	Wave 6.1 6.2 6.3 6.4 6.5	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru- Using Oceani Couple 6.5.1 6.5.2	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Llized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves ed KdV Equations for a Laboratory Experiment. Solution of the Sturm–Liouville Problem. Initial Problem Formulation	119 119 122 123 124 124 126 127 129 133 138 138 138 138
6	Wave 6.1 6.2 6.3 6.4 6.5 6.6	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru- Using Oceani Couple 6.5.1 6.5.2 Numer	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Long Atmospheric Internal Waves Ilized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves ed KdV Equations for a Laboratory Experiment. Solution of the Sturm–Liouville Problem. Initial Problem Formulation	119 119 122 123 123 124 124 124 126 127 129 133 138 138 138
6	Wave 6.1 6.2 6.3 6.4 6.5 6.6	eguide F Fundar 6.1.1 6.1.2 6.1.3 Genera 6.2.1 6.2.2 6.2.3 Constru- Using Oceani Couple 6.5.1 6.5.2 Numer 6.6.1	Propagation in Hydrodynamics nentals of Hydrothermodynamics Determination of Wave Type Atmospheric Acoustic and Internal Gravity Waves Long Atmospheric Internal Waves Long Atmospheric Internal Waves Ilized Fluid Dynamics Introductory Notes Six-Momentum Bhatnagar–Gross–Krook Model Piecewise Continuous Distribution Function. Closure of the System ucting Solutions of the Fluid Dynamics System the WKB Method c Internal Gravity Waves ed KdV Equations for a Laboratory Experiment Physical Model of the McEwan Experiment Solution of the Sturm–Liouville Problem. Initial Problem Formulation Gal Method Gal Method	119 119 122 123 123 124 124 124 126 127 129 133 138 138 138 138

8

9

	6.7	Planetary Waves	142
		6.7.1 Equations for Barotropic Waves on a Sphere	142
		6.7.2 Projection Operators as a Diagnostic Tool	145
	6.8	Planetary Waves in the β -Plane Model	146
		6.8.1 Basic Equations and Waveguide Mode Expansion	146
		6.8.2 Projection Operators for Rossby and Poincaré	
		Waves	150
	6.9	Application of Projection Operators to Planetary Waves	
		in the Upper Atmosphere	152
	6.10	Finite-Difference Analogues of Projection Operators	158
	6.11	Appendix Ia. A Limiting Case of Large Collision	
		Frequencies	160
	6.12	Appendix Ib. Free Molecular Flow Limit (Collisionless	
		Regime)	161
	6.13	Appendix II: Stability Proof of the Finite-Difference	
		Scheme for CKdV	164
	6.14	Appendix III: Proof of Convergence of the Scheme	166
	Refer	ences	169
7	Guid	e Propagation and Interaction of Plasma Waves.	
,	Meta	materials	173
	7.1	Plasma Wayes in a Stratified Plasma	173
		7.1.1 Maxwell System	173
		7.1.2 Vlasov Plasma	174
	7.2	Weakly Inhomogeneous Plasmas	180
	7.3	Interaction of Plasma Waves	182
		7.3.1 Langmuir and Ion Wave Interaction	182
	7.4	Plasma Waveguides	185
		7.4.1 Plasma Confinement	185
		7.4.2 Perturbations in Confined Plasmas	187
		7.4.3 Approach Using the Hydrodynamic. Flute	
		Instability	188
	7.5	Helicoidal Wave Interactions	190
		7.5.1 Basic Equations	191
		7.5.2 Introducing Projectors P_+ and P	193
		7.5.3 Model with Nonlinear Term. Three-Wave	
		Equation	194
		7.5.4 Three-Wave System in the 1+1 Case	196
	7.6	Twofold Darboux Transformation (TfDT) in the Description	
		of the Three-Wave System	196
		7.6.1 Solutions Derived Using the TfDT	197
		7.6.2 Plots of the Solutions and Discussion	198
		7.6.3 Kinetic Description of Plasma Waves	
		and Turbulence	198

7.7	Electro 7.7.1	Demagetic Waves in a Metamaterial Dispersion in a 1D Metamaterial	200 200	
	7.7.2	Rectangular Waveguide Filled with Metamaterial	200	
Refe	rences .		204	
N T		*1 101 1 337		
	owavegu		207	
8.1	Genera	al Remarks	207	
8.2	the Su	rfical Tube with Point Potentials Centered at	200	
		Geometry	209	
	0.2.1 8.2.2	Summetry and Black Eurotians	209	
83	0.2.2 Metho	d of Zaro Pango Dotantials for the Plach Eurotians	209	
0.5	831	Resigned and Resig	212	
	837	Eleguet and Wigner Theorems	212	
	833	Discrete Spectrum for the Finite Tube	213	
	8.3.5	Infinite Tube, Rands	217	
84	Dressi	number 1 abe. Datas: $1 \neq 0$ and Accounting	219	
0.7	Dressing of Zero-Range Potentials for $l \neq 0$ and Accounting for Spin			
	841	Darboux Transformation and Its Application	220	
	842	Accounting for the Spin Variable	220	
85	Bloch	Wave Scattering by a 7RP	224	
0.5	8 5 1	Flux Normalization	227	
	8.5.2	Scattering of Bloch Electrons by a ZRP	221	
	0.012	Formulation of the Problem	228	
8.6	Case S	Study: $M = 1$, a Cyclic Molecule	229	
	8.6.1	Bound State Problem	229	
	8.6.2	Continuous Spectrum. Scattering Problem	231	
	8.6.3	Results for the Benzene Molecule	233	
Refe	rences .		236	
Miss				
	owavegi	Generation Concerned Transport	239	
9.1	Dasic V	hang Natural Dunamics	239	
9.2		Compared Continuum LL Exceptions	241	
	9.2.1	LLC Equation for Microwing	241	
0.2	9.2.2	Theory	243	
9.5		Walken Solution	244	
	9.3.1	Instability of the Welker Solution	244	
	9.3.2	Nenoviras as Usisenberg Chains	244	
0.4	Propag	Nation of a Domain Wall in a Culindrical Amarchaus	247	
7.4	Propagation of a Domain Wall in a Cylindrical Amorphous			
	9 <u>/</u> 1	Introductory Remarks	241 217	
	9.7.1	LLG Transforms Statement of the Drohlam	241 210	
	943	The Search for Particular Solutions	240 240	
	J. T .J		249	

		9.4.4	Additional Assumptions. Explicit Form	
			of the Solution	250
		9.4.5	Examples of DWs in 3D	251
		9.4.6	Velocity of DW Propagation. Determining	
			the Anisotropy Constants	253
	Refer	ences	•••••••••••••••••••••••••••••••••••••••	255
10	Kine	tics of C	harges in Waveguides. Charge Transport	259
	10.1	Formul	ation of the Kinetic Problem	259
		10.1.1	Motivation	259
		10.1.2	Kinetic Equation and Boundary Conditions	260
		10.1.3	Distribution Averaging. Electric Current as Particle	
			Number Rate	261
	10.2	Method	l of Solution. N-Fold Scattering Expansion	262
		10.2.1	Cylindrical Coordinates	262
		10.2.2	Iterative Construction	264
	10.3	Station	ary Case	265
		10.3.1	Boundary Regime Problem	265
		10.3.2	Alternative Expansion for the Stationary Case	265
		10.3.3	Stationary Solution for an Empty Cylinder	
			with Reflecting Wall	266
	10.4	Kinetic	Model of Electron Transport in a Cylindrical	
		Nanow	ire with a Rough Surface	270
		10.4.1	Main Goal	270
		10.4.2	Scattering Models	271
		10.4.3	Kinetic Equation	273
		10.4.4	Rough Boundary Case	274
		10.4.5	Fourier Transform Solution	274
		10.4.6	Electron Transport	276
	Refer	ences	····	279
Ind	ex			281