## Contents

1.	1. Introduction 1				
2	The Scalar Waye Field				
2.	2.1 Equations of Geometrical Ontics			ž	
	2.1	211	Mathematical Background	ž	
		2.1.1	Field Expansion in a Dimensionless Parameter	5	
		2.1.2	Field Expansion in Inverse Wave Numbers	7	
		2.1.3 2.1.4	Initial Conditions for the Eikonal	,	
		2.1.4	and Amplitude Equations	8	
		215	Asymptotic Nature of the Pay Series	8	
	22	Rave	and the Fikonal	ğ	
	2.2	221	The Method of Characteristics	ģ	
		2.2.1	Ray Equations and the Fikonal	ú	
		2.2.2	Curvature and Torsion of Rave	13	
		2.2.3	Initial Conditions for Rays Ray Coordinates	14	
		2.2.4	Ray Families and Phase Fronts	15	
		2.2.5	The Fermat Principle	17	
		2.2.0	Ray Equations in Curvilinear Coordinates	18	
		2.2.7	Other Types of Ray	20	
	23	Wave	Amplitude	22	
	2.5	231	Formal Solution of the Transport Fountion	22	
		232	Rays and the Direction of Energy Flow	23	
		233	Conservation of Energy Flux in a Ray Tube	24	
		234	The Field due to a Point Source		
		2.3.1	in an Inhomogeneous Medium	26	
		235	The Resultant Field in the Ray-Optics Approximation	26	
		236	The Field Amplitudes of Higher-Order Approximations	28	
		237	Accounting for Weak Absorption	28	
	24	Caust	ics	29	
		2.4.1	Fundamental Properties	29	
		2.4.2	Wave-Field Focusing on Caustics	31	
		2.4.3	Types of Caustics	32	
		2.4.4	Structurally Stable and Unstable Caustics	•	
			in Physical Problems	38	
		2.4.5	Other Types of Caustics	39	
		2.4.6	Singularities of Phase Fronts	39	
		2.4.7	Phase Shifts at Caustics	40	
	2.5	Reflection and Refraction of Wayes at Interfaces			
		2.5.1	The Locality Principle in Wave Reflection	41	
			,, <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> _, <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> _, <b></b> , <b></b> , <b></b> _, <b></b> , <b></b> _, <b></b> ,		

	2.5.2	Relations for Rays and Eikonals	42
	2.5.3	Reflection Formulas for Amplitude	43
	2.5.4	Reflection from Weak Interfaces	45
	2.5.5	The Geometrical Optics of Surface Waves	46
2.6	Recit	procity of Rays and Caustics	47
	2.6.1	The Reciprocity Theorem	47
	2.6.2	Reciprocity Relations for Rays and Caustics	47
2.7	Space	-Time Geometrical Optics	49
	2.7.1	The Wave Equation for Media	
		with Temporal (Frequency) Dispersion	49
	2.7.2	Necessary Conditions for the Geometrical-Optics	
		Applied to Quasi-Monochromatic Wave Packets	50
	2.7.3	Differential Form of the Constitutive Equation (2.7.2)	51
	2.7.4	Eikonal and Transport Equations	52
	2.7.5	Space-Time Rays	53
	2.7.6	Initial Conditions	55
	2.7.7	Eikonal and Wave Amplitude	57
	2.7.8	Space-Time Caustics	60
	2.7.9	Propagation of Field Discontinuities	
	_	in Nondispersive Media	61
2.8	Separ	ation of Variables in the Eikonal Equation	64
	2.8.1	The Complete Integral of the Eikonal Equation	64
	2.8.2	Separation of Variables in Two Dimensions	
		(Cartesian Coordinates)	65
	2.8.3	Separation of Variables in Two Dimensions	
		(Curvilinear Orthogonal Coordinates)	66
	2.8.4	Separation of Variables in Three-Dimensional Space	68
	2.8.5	Incomplete Separation of Variables	70
• •	2.8.6	The Complete Integral of Eikonal and Ray Equations	71
2.9	Pertu	rbation Techniques	
	for G	eometrical-Optics Equations	73
	2.9.1	The Perturbation Method for the Eikonal	73
	2.9.2	The Perturbation Method for Rays	74
	2.9.3	Perturbations in Homogeneous Media	75
<b>a</b> 10	2.9.4	Perturbations in Nonhomogeneous Media	77
2.10	Appli	cability of Geometrical Optics	79
	2.10.1	Existent Estimators of Method's Errors	79
	2.10.2	Fresnel Zones and Fresnel Volume of Rays	
	• • • •	in Inhomogeneous Media	80
	2.10.3	The Physical Meaning of the Ray	87
	2.10.4	Heuristic Criteria on Geometrical-Optics Applicability	88
	2.10.5	Applicability Conditions	
	• • • •	for Space-Time Geometrical Optics	89
	2.10.6	Heuristic Accuracy Estimates of Geometrical Optics	93
	2.10.7	Estimating the Width of a Caustic Zone	93
	2.10.8	Indistinguishability of Rays in the Caustic Zone	96
	2.10.9	Observability of Caustics	96
	2.10.1	U Field Estimations Beyond the Validity Region	<u> </u>
		or Geometrical Optics	97

		2.10.11 Field-Focusing Indices at Caustics			
		2.10.12 Stability with Respect to Small Perturbations			
		2.10.1	3 Wave-Pattern Analysis in General	. 99	
3.	Ap	pplications of the Ray Methods			
	3.1	Wave	s in Homogeneous Media	101	
		3.1.1	Rays and the Eikonal	101	
		3.1.2	The Wave Amplitude	102	
		3.1.3	Caustics	104	
		3.1.4	The Plane Phase-Amplitude Screen	105	
		3.1.5	The Sinusoidal Phase Screen. An Illustrative Example	107	
		3.1.6	Applicability Conditions for Geometrical Optics	109	
		3.1.7	Geometrical Optics in Far and Near Antenna Fields.		
			Wave Beam Propagation	110	
		3.1.8	On the Phase Center of an Antenna or a Scatterer	113	
		3.1.9	Field Near a Lens Focus	114	
		3.1.10	) Field at the Focus of a Lens		
			with Cylindrical (Spherical) Aberration	116	
	3.2	Refle	ction and Refraction at an Interface		
		Betwe	een Homogeneous Media	121	
		3.2.1	Reflection Formulas	121	
		3.2.2	Divergence of Reflected and Refracted Rays	121	
		3.2.3	Effective Scattering Surface of a Body		
			in the Geometrical-Optics Approximation	125	
		3.2.4	Reflection Far Field of a Directional Point Source	125	
		3.2.5	Caustics of Refracted and Reflected Rays	126	
		3.2.6	Examples of Catacaustics and Diacaustics	127	
		3.2.7	Applicability of Reflection Formulas	128	
		3.2.8	The Invalidity Domain in the Vicinity of a Tangent Ray	131	
		3.2.9	Wave Diffraction at a Surface of Variable Impedance	131	
	3.3	Rays	and Caustics in Plane-Stratified Media	132	
		3.3.1	Ray Equations	132	
		3.3.2	Ray Tracing in a Plane-Stratified Medium	135	
		3.3.3	Equations of Caustics, and the Geometry		
			of the Ray Family	136	
		3.3.4	Rays and Caustics due to a Point Source		
			in an Inhomogeneous Medium	137	
		3.3.5	Rays and Caustics in a Linear Layer	139	
		3.3.6	Layers of Other Profiles	142	
		3.3.7	Plane Waves in a Parabolic Layer	143	
		3.3.8	A Point Source in a Parabolic Laver	145	
	3.4	Wave	Fields in Plane-Stratified Media	146	
		3.4.1	The Field of an Arbitrary Wave	146	
		3.4.2	The Field of a Plane Wave	148	
		3.4.3	Fields of Point and Linear Sources	150	
		3.4.4	A Point Source in a Linear Laver	152	
		3.4.5	A Point Source in a Parabolic Laver	155	
		3.4.6	The Fresnel Volumes in Plane-Stratified Media	156	

IX

	3.4.7 Validity Conditions			
	of the Geometrical-Optics Approximation	159		
3.5	Waves in Radially Inhomogeneous Media			
	3.5.1 Ray Equations for Spherically Stratified Media	160		
	3.5.2 The Eikonal Function for Spherically Stratified Media .	163		
	3.5.3 Cylindrically Stratified Media	164		
	3.5.4 Ray Geometry	165		
	3.5.5 The Field due to a Point Source	167		
	3.5.6 The Field of a Plane Wave	169		
	3.5.7 Caustics	170		
3.6	Tapered and Other Inhomogeneous Media	172		
	3.6.1 The Eikonal and Rays in a Tapered Medium	172		
	3.6.2 The Field of a Plane Wave	174		
	3.6.3 The Field due to a Linear Source	175		
	3.6.4 Ray Equations in a Two-Dimensional Medium			
	with a Special Profile	176		
	3.6.5 The Field of a Point Source			
	(Axially Symmetric Problem)	178		
	3.6.6 A Plane Wave Incident			
	on the Two-Dimensionally Inhomogeneous Medium	180		
	3.6.7 Weakly Inhomogeneous, Ouasi-Stratified.			
	and Random Media	181		
3.7	Geometrical Optics of Waveguides and Resonators	182		
	3.7.1 Geometrical Optics of Waveguides	182		
	3.7.2 Ray Description of Modes in Uniform Waveguides	184		
	3.7.3 Adiabatic Modes of Smoothly Nonuniform Waveguides	187		
	3.7.4 Ionospheric Wave Channels.			
	The Adiabatic Invariant Method	188		
	3.7.5 Underwater-Sound Ducts	100		
	Summing-Un Incoherent Wave Fields	190		
	3.7.6 Optical Fibers	190		
	3.7.7 Mode Conversion in Smoothly Nonuniform Waveguides	192		
	3.7.8 Normal Modes in Cavity Resonators	192		
3.8	Wave Scattering at Localized Inhomogeneities	192		
2.0	3.8.1 Effective Scattering Surface	192		
	3.8.2 Scattering by a Body in an Inhomogeneous Medium	196		
	383 Effective Scattering Surface	170		
	of a Spherically Stratified Inhomogeneity	197		
	384 Effective Scattering Surface of a Perfectly Conducting	177		
	Solution Scherically Stratified Medium	100		
	3.8.5 Effective Scattering Surface of a Specific	1))		
	Two-Dimensionally Inhomogeneous Formation	201		
	3.8.6 Scattering of a Subarical Wave by a Localized	201		
	Inhomogeneity	202		
	387 Scattering by Week Localized Inhomogeneities	202		
30	Dulse Propagation	204		
5.7	391 General Relations for the Dissma (Childed)	203		
	Dispersion Law	205		
		203		

		3.9.2	A Homogeneous Medium with an Arbitrary	
			Dispersion Law	206
		3.9.3	A Plane, Frequency-Modulated Pulse	
			in a Homogeneous Medium	207
		3.9.4	Dispersive Compression of FM Pulses	
			in Homogeneous Media	211
		3.9.5	Plane-Stratified Dispersive Media	213
		3.9.6	Near and Far Fields of a Pulse	214
	3.10	Nume	rical Methods in the Geometrical Optics	
		of Inh	omogeneous Media	216
		3.10.1	The Ray-Tracing Analysis	216
		3.10.2	Computing the Eikonal and Wave Amplitude	218
		3.10.3	Problems of Numerical Analysis	221
	3.11	Invers	e Problems of Geometrical Optics	222
		3.11.1	Reflection and Refraction at Interfaces	222
		3.11.2	Inverse Problems for Given Models	
			of the Inhomogeneous Medium	223
		3.11.3	Multidimensional Inverse Problems	224
		3.11.4	Nonstationary Inverse Problems	225
4.	Vec	tor Wa	ave Fields	226
	4.1	Trans	verse Electromagnetic Waves in Isotropic Media	226
		4.1.1	Maxwell Equations for Monochromatic Waves	226
		4.1.2	The Debye Expansion and the Iterative Equations	227
		4.1.3	The Eikonal Equation	227
		4.1.4	Transverse Nature of Zero Approximation Waves.	
			Polarization Degeneracy	228
		4.1.5	Consistency of the First-Approximation Equations	229
		4.1.6	Conserving Energy Flow in a Ray Tube	230
		4.1.7	Preserving the Polarization Ellipse	230
		4.1.8	Rotation of Field Vectors (Rytov's Law)	231
		4.1.9	Polarization of Transverse Waves	233
		4.1.10	Longitudinal Components of the Field	233
		4.1.11	Reflection of Transverse Electromagnetic Waves	
			from Interfaces	234
		4.1.12	Polarization Degeneracy in Problems of Quantum	
			Mechanics and Theory of Elasticity	234
	4.2	Indep	endent Normal Waves in an Anisotropic Medium	235
		4.2.1	Equation of the Eikonal	235
		4.2.2	Independent Normal Mode	236
		4.2.3	Ray Equations	237
		4.2.4	Solving the Eikonal Equation	239
		4.2.5	Definition of Mode Polarization Vectors	240
		4.2.6	Consistency of Equations of the First Approximation .	240
		4.2.7	The Transfer Equation	241
		4.2.8	Equation for the Argument of a Complex Amplitude	242
		4.2.9	Rays and Energy Paths.	
			The Fresnel Volumes in Anisotropic Media	243
			•	

	4.2.10	An Account of Weak Absorption	244	
	4.2.11 Reflection and Refraction at the Boundaries			
		of Anisotropic Media	245	
	4.2.12	2 Some Specific Results	246	
	a) The Field of a Point Source			
		in an Anisotropic Medium	247	
		b) Wayes in Plane-Stratified Anisotropic Media	247	
		c) Separation of Variables in the Equation of Eikonal		
		in the General Case	249	
		d) Perturbation Theory	249	
	4.2.13	Divergence of First-Approximation Fields		
		at Polarization Degeneracy	249	
	4214	Other Vector Problems	251	
43	Intera	action of Normal Modes in Inhomogeneous	201	
4.5	Anisc	atronic Media	251	
	121	Wayes in Weakly Anisotronic Media	231	
	4.5.1	The Ouesi Isotronia Anneximation	251	
	422	Different Forme of the Doubtions	251	
	4.3.2	Different Forms of the Equations	252	
	4 9 9	of the Quasi-Isotropic Approximation	255	
	4.3.3	Solution Techniques		
		for the Quasi-Isotropic Approximation	255	
	4.3.4	On the Error of the Quasi-Isotropic Approximation	255	
	4.3.5	Applications of the Quasi-Isotropic Approximation	256	
	4.3.6	The Quasi-Degenerate Approximation		
		of Geometrical Optics	257	
4.4	Equations of Geometrical Optics for Nonharmonic Electro-			
	magnetic Waves in the General Case of Inhomogeneous			
	and N	Vonstationary Dispersive Media	258	
	4.4.1	The Maxwell Equations in Inhomogeneous and Non-		
		stationary Media of Temporal and Spatial Dispersion	258	
	4.4.2	The Constitutive Equation in Differential Form	260	
	4.4.3	Equations for the Fields in Zeroth and First		
		Approximations	261	
	4.4.4	The Eikonal Equation. Space-Time Rays	262	
	4.4.5	The Transfer Equation for Independent Normal Modes		
		in an Anisotropic Medium	263	
	4.4.6	The Group Velocity Theorem	264	
	4.4.7	Integrating the Transfer Equation		
		Along Space-Time Rays	266	
	4.4.8	Transverse Modes in an Isotropic Medium	267	
	4.4.9	Longitudinal Wayes in an Isotropic Medium	269	
	4410	Wayes in Weakly Anisotronic Media	270	
4.5	Const	itutive Equations for Nonstationary and Inhomogeneous	2.0	
	Diena	rsive Media Existence of Adiabatic Invariance	270	
	451	Corrections to the Augei-Stationary Dermittivity Tansor	270	
	4.5.1	Physical Phanomena Due to the Deviction of a	210	
	<del>4</del> .J.Z	from Its Oussi Stationary Value	271	
	152	Existence of the Adjubatic Investigat	271	
	4.3.3	Existence of the Autabatic Invariant	212	

	4.5.4	Phenomenological Evaluation of the Anti-Hermitian Part of the Correction for the Ouasi-Stationary			
		Permittivity Tensor in Transparent Media	273		
4	4.6 Wave	Processes in Nonstationary Media	274		
	4.6.1	One-Dimensional Problem. General Relationships	274		
	4.6.2	Nonstationary Nondispersive Media	276		
	4.6.3	Nonstationary Dispersive Media	279		
	4.6.4	Evolution of Short Pulses	281		
	4.6.5	Reflection from Moving Interfaces	282		
	4.6.6	Perturbation Theory in Nonstationary Problems	286		
5. (	Conclusio	nn	287		
Ref	erences		291		
Subject Index					