## **Contents**

1. Introduction				
2.	The	Scala	r Wave Field	3
_•			tions of Geometrical Optics	3
			Mathematical Background	1
			Field Expansion in a Dimensionless Parameter	4
			Field Expansion in Inverse Wave Numbers	•
			Initial Conditions for the Eikonal	
			and Amplitude Equations	8
		2.1.5	Asymptotic Nature of the Ray Series	8
	2.2	Rays	and the Eikonal	9
		2.2.1	The Method of Characteristics	9
		2.2.2	Ray Equations and the Eikonal	1
		2.2.3	Curvature and Torsion of Rays	1.
		2.2.4	Initial Conditions for Rays. Ray Coordinates	14
		2.2.5	Ray Families and Phase Fronts	
		2.2.6	The Fermat Principle	1
			Ray Equations in Curvilinear Coordinates	
			Other Types of Ray	
	2.3		Amplitude	
			Formal Solution of the Transport Equation	
			Rays and the Direction of Energy Flow	
			Conservation of Energy Flux in a Ray Tube	2
		2.3.4	The Field due to a Point Source	
			in an Inhomogeneous Medium	
			The Resultant Field in the Ray-Optics Approximation .	
			The Field Amplitudes of Higher-Order Approximations .	
			Accounting for Weak Absorption	
	2.4	Caust	ics	2
		2.4.1	Fundamental Properties	2
		2.4.2	Wave-Field Focusing on Caustics	3
		2.4.3	Types of Caustics	3
		2.4.4	Structurally Stable and Unstable Caustics	
			in Physical Problems	
			Other Types of Caustics	
			Singularities of Phase Fronts	
			Phase Shifts at Caustics	
	2.5		ection and Refraction of Waves at Interfaces	
		2.5.1	The Locality Principle in Wave Reflection	4

	2.5.2 Relations for Rays and Eikonals	42	2.10.11 Field-Focusing Indices at Caustics	
	2.5.3 Reflection Formulas for Amplitude	43	2.10.12 Stability with Respect to Small Perturbations	9
	2.5.4 Reflection from Weak Interfaces	45	2.10.13 Wave-Pattern Analysis in General	99
	2.5.5 The Geometrical Optics of Surface Waves	46		
26	Reciprocity of Rays and Caustics	47	3. Applications of the Ray Methods 1	10
2.0	2.6.1 The Reciprocity Theorem	47	3.1 Waves in Homogeneous Media	10
	2.6.2 Reciprocity Relations for Rays and Caustics	47	3.1.1 Rays and the Eikonal	
27	Space-Time Geometrical Optics	49	3.1.2 The Wave Amplitude	
2.1	2.7.1 The Wave Equation for Media	• •	3.1.3 Caustics	
	with Temporal (Frequency) Dispersion	49	3.1.4 The Plane Phase-Amplitude Screen	
	2.7.2 Necessary Conditions for the Geometrical-Optics	• •	3.1.5 The Sinusoidal Phase Screen. An Illustrative Example . 1	
	Applied to Quasi-Monochromatic Wave Packets	50		109
	2.7.3 Differential Form of the Constitutive Equation (2.7.2)		3.1.7 Geometrical Optics in Far and Near Antenna Fields.	
	2.7.4 Eikonal and Transport Equations		Wave Beam Propagation	110
	2.7.5 Space-Time Rays		3.1.8 On the Phase Center of an Antenna or a Scatterer 1	
	2.7.6 Initial Conditions		3.1.9 Field Near a Lens Focus	
	2.7.7 Eikonal and Wave Amplitude		3.1.10 Field at the Focus of a Lens	
	2.7.8 Space-Time Caustics		with Cylindrical (Spherical) Aberration	1 17
	2.7.9 Propagation of Field Discontinuities	00	3.2 Reflection and Refraction at an Interface	1 4 1
	in Nondispersive Media	61	Between Homogeneous Media	12
28	Separation of Variables in the Eikonal Equation		3.2.1 Reflection Formulas	12
2.0	2.8.1 The Complete Integral of the Eikonal Equation		3.2.2 Divergence of Reflected and Refracted Rays	
	2.8.2 Separation of Variables in Two Dimensions	04	3.2.3 Effective Scattering Surface of a Body	
	(Cartesian Coordinates)	65	in the Geometrical-Optics Approximation	124
	2.8.3 Separation of Variables in Two Dimensions	03	3.2.4 Reflection Far Field of a Directional Point Source 1	12.
		66	3.2.5 Caustics of Refracted and Reflected Rays	
	(Curvilinear Orthogonal Coordinates)	00	3.2.6 Examples of Catacaustics and Diacaustics	
	2.8.4 Separation of Variables in Three-Dimensional Space		A A M	
	2.8.5 Incomplete Separation of Variables		3.2.7 Applicability Demain in the Visinity of a Tongant Dev. 1	128
20	2.8.6 The Complete Integral of Eikonal and Ray Equations	/1	3.2.8 The Invalidity Domain in the Vicinity of a Tangent Ray 1	13
2.9	Perturbation Techniques	72	3.2.9 Wave Diffraction at a Surface of Variable Impedance . 1	13
	for Geometrical-Optics Equations	73	3.3 Rays and Caustics in Plane-Stratified Media	
	2.9.1 The Perturbation Method for the Eikonal		3.3.1 Ray Equations	
	2.9.2 The Perturbation Method for Rays	74	3.3.2 Ray Tracing in a Plane-Stratified Medium	13:
	2.9.3 Perturbations in Homogeneous Media	75	3.3.3 Equations of Caustics, and the Geometry	
2.10	2.9.4 Perturbations in Nonhomogeneous Media	77	of the Ray Family	136
2.10	Applicability of Geometrical Optics	79	3.3.4 Rays and Caustics due to a Point Source	
	2.10.1 Existent Estimators of Method's Errors	79	in an Inhomogeneous Medium	137
	2.10.2 Fresnel Zones and Fresnel Volume of Rays		3.3.5 Rays and Caustics in a Linear Layer	139
	in Inhomogeneous Media	80	3.3.6 Layers of Other Profiles	142
	2.10.3 The Physical Meaning of the Ray	87	3.3.7 Plane Waves in a Parabolic Layer	143
	2.10.4 Heuristic Criteria on Geometrical-Optics Applicability	88	3.3.8 A Point Source in a Parabolic Layer	14:
	2.10.5 Applicability Conditions		3.4 Wave Fields in Plane-Stratified Media	
	for Space-Time Geometrical Optics	89	3.4.1 The Field of an Arbitrary Wave	
	2.10.6 Heuristic Accuracy Estimates of Geometrical Optics	93	3.4.2 The Field of a Plane Wave	
	2.10.7 Estimating the Width of a Caustic Zone	93	3.4.3 Fields of Point and Linear Sources	
	2.10.8 Indistinguishability of Rays in the Caustic Zone		3.4.4 A Point Source in a Linear Layer	152
	2.10.9 Observability of Caustics	96	3.4.5 A Point Source in a Parabolic Layer	15:
	2.10.10 Field Estimations Beyond the Validity Region		3.4.6 The Fresnel Volumes in Plane-Stratified Media 1	150
	of Geometrical Optics	97		

	2.4.7 Validity Conditions		3.9.2 A Homogeneous Medium with an Arbitrary	
	3.4.7 Validity Conditions of the Geometrical-Optics Approximation	159	Dispersion Law	206
2 5	Waves in Radially Inhomogeneous Media		3.9.3 A Plane, Frequency-Modulated Pulse	
3.3	3.5.1 Ray Equations for Spherically Stratified Media		in a Homogeneous Medium	207
	3.5.2 The Eikonal Function for Spherically Stratified Media		3.9.4 Dispersive Compression of FM Pulses	
	3.5.3 Cylindrically Stratified Media		in Homogeneous Media	211
	3.5.4 Ray Geometry		3.9.5 Plane-Stratified Dispersive Media	
	3.5.5 The Field due to a Point Source		3.9.6 Near and Far Fields of a Pulse	
	3.5.6 The Field of a Plane Wave		3.10 Numerical Methods in the Geometrical Optics	
	3.5.7 Caustics		of Inhomogeneous Media	216
3.6	Tapered and Other Inhomogeneous Media		3.10.1 The Ray-Tracing Analysis	
3.0	3.6.1 The Eikonal and Rays in a Tapered Medium		3.10.2 Computing the Eikonal and Wave Amplitude	
	3.6.2 The Field of a Plane Wave		3.10.3 Problems of Numerical Analysis	
	3.6.3 The Field due to a Linear Source		3.11 Inverse Problems of Geometrical Optics	
	3.6.4 Ray Equations in a Two-Dimensional Medium	175	3.11.1 Reflection and Refraction at Interfaces	
	with a Special Profile		3.11.2 Inverse Problems for Given Models	
	3.6.5 The Field of a Point Source	170	of the Inhomogeneous Medium	223
	(Axially Symmetric Problem)	179	3.11.3 Multidimensional Inverse Problems	
	3.6.6 A Plane Wave Incident	170	3.11.4 Nonstationary Inverse Problems	
	on the Two-Dimensionally Inhomogeneous Medium	190	The state of the s	
	3.6.7 Weakly Inhomogeneous, Quasi-Stratified,	100	4. Vector Wave Fields	226
	and Random Media	181	4.1 Transverse Electromagnetic Waves in Isotropic Media	
37	Geometrical Optics of Waveguides and Resonators		4.1.1 Maxwell Equations for Monochromatic Waves	
3.7	3.7.1 Geometrical Optics of Waveguides		4.1.2 The Debye Expansion and the Iterative Equations	
	3.7.2 Ray Description of Modes in Uniform Waveguides		4.1.3 The Eikonal Equation	
	3.7.3 Adiabatic Modes of Smoothly Nonuniform Waveguides	187	4.1.4 Transverse Nature of Zero Approximation Waves.	
	3.7.4 Ionospheric Wave Channels.	107	Polarization Degeneracy	228
	The Adiabatic Invariant Method	188	4.1.5 Consistency of the First-Approximation Equations	
	3.7.5 Underwater-Sound Ducts.	100	4.1.6 Conserving Energy Flow in a Ray Tube	
	Summing-Up Incoherent Wave Fields	100	4.1.7 Preserving the Polarization Ellipse	
	3.7.6 Optical Fibers		4.1.8 Rotation of Field Vectors (Rytov's Law)	
	3.7.7 Mode Conversion in Smoothly Nonuniform Waveguides		4.1.9 Polarization of Transverse Waves	
	3.7.8 Normal Modes in Cavity Resonators		4.1.10 Longitudinal Components of the Field	
3 8	Wave Scattering at Localized Inhomogeneities		4.1.11 Reflection of Transverse Electromagnetic Waves	
5.0	3.8.1 Effective Scattering Surface		from Interfaces	234
	3.8.2 Scattering by a Body in an Inhomogeneous Medium		4.1.12 Polarization Degeneracy in Problems of Quantum	25
	3.8.3 Effective Scattering Surface	170	Mechanics and Theory of Elasticity	234
	of a Spherically Stratified Inhomogeneity	107	4.2 Independent Normal Waves in an Anisotropic Medium	
	3.8.4 Effective Scattering Surface of a Perfectly Conducting	177	4.2.1 Equation of the Eikonal	
	Sphere in a Spherically Stratified Medium	199	4.2.2 Independent Normal Mode	
	3.8.5 Effective Scattering Surface of a Specific	199	4.2.3 Ray Equations	
	Two-Dimensionally Inhomogeneous Formation	201	4.2.4 Solving the Eikonal Equation	
	3.8.6 Scattering of a Spherical Wave by a Localized	201	4.2.5 Definition of Mode Polarization Vectors	
		202	4.2.6 Consistency of Equations of the First Approximation .	
	Inhomogeneity		4.2.7 The Transfer Equation	
30	Pulse Propagation		4.2.8 Equation for the Argument of a Complex Amplitude .	
3.7	3.9.1 General Relations for the Plasma (Guided)	203	4.2.9 Rays and Energy Paths.	272
	Dispersion Law	205	The Fresnel Volumes in Anisotropic Media	243
	Disposition	203	The Fresher volumes in Amsouropic Media	473

	4.2.10	An Account of Weak Absorption	244		
	4.2.11	Reflection and Refraction at the Boundaries			
		of Anisotropic Media	245		
	4.2.12	Some Specific Results	246		
		a) The Field of a Point Source			
		in an Anisotropic Medium	247		
		b) Waves in Plane-Stratified Anisotropic Media	247		
		c) Separation of Variables in the Equation of Eikonal			
		in the General Case	249		
		d) Perturbation Theory	249		
	4.2.13	Divergence of First-Approximation Fields			
		at Polarization Degeneracy	249		
	4.2.14	Other Vector Problems	251		
4.3		action of Normal Modes in Inhomogeneous			
		ptropic Media	251		
	4.3.1	Waves in Weakly Anisotropic Media.			
		The Quasi-Isotropic Approximation	251		
	4.3.2		231		
	4.5.2	of the Quasi-Isotropic Approximation	253		
	4.3.3	Solution Techniques	233		
	7.5.5	for the Quasi-Isotropic Approximation	255		
	4.3.4	The state of the s	255		
		Applications of the Quasi-Isotropic Approximation	256		
	4.3.6	The Quasi-Degenerate Approximation	230		
	4.5.0	of Geometrical Optics	257		
1.4	Faus	tions of Geometrical Optics for Nonharmonic Electro-	231		
7.7		etic Waves in the General Case of Inhomogeneous			
		Nonstationary Dispersive Media	258		
	4.4.1		230		
	7.7.1	stationary Media of Temporal and Spatial Dispersion	258		
	4.4.2	The Constitutive Equation in Differential Form	260		
	4.4.3	Equations for the Fields in Zeroth and First	200		
	4.4.3		261		
	444	Approximations	262		
	4.4.5	The Eikonal Equation Space-Time Rays	202		
	4.4.3	The Transfer Equation for Independent Normal Modes	263		
	116	in an Anisotropic Medium	264		
	4.4.6	The Group Velocity Theorem	204		
	4.4.7	Integrating the Transfer Equation	200		
	4 4 0	Along Space-Time Rays	266		
	4.4.8	Transverse Modes in an Isotropic Medium	267		
	4.4.9		269		
		Waves in Weakly Anisotropic Media	270		
1.5	Constitutive Equations for Nonstationary and Inhomogeneous				
		rsive Media. Existence of Adiabatic Invariance	270		
	4.5.1	Corrections to the Quasi-Stationary Permittivity Tensor	270		
	4.5.2	Physical Phenomena Due to the Deviation of $\epsilon_{\alpha\beta}$			
	4.5.0	from Its Quasi-Stationary Value	271		
	411	Existence of the Adiabatic Invariant	272		

4.:	5.4 Phenomenological Evaluation of the Anti-Hermitian	
	Part of the Correction for the Quasi-Stationary	
	Permittivity Tensor in Transparent Media	273
4.6 W	ave Processes in Nonstationary Media	274
4.0	6.1 One-Dimensional Problem. General Relationships	274
4.0	6.2 Nonstationary Nondispersive Media	276
4.0	6.3 Nonstationary Dispersive Media	279
4.0	6.4 Evolution of Short Pulses	281
4.	6.5 Reflection from Moving Interfaces	282
4.	6.6 Perturbation Theory in Nonstationary Problems	286
5. Conclu	usion	287
Referenc	es	291
Subject I	nder	311