

Contents

Part I Characterization of Complex Systems

1 Primer on Complex Systems	3
1.1 Introduction	3
1.1.1 What Is a Complex System?	3
1.1.2 Examples of Complex Systems	5
1.1.3 Complex Systems in Plasma Science	7
1.1.4 Complexity Science	10
1.2 Key Concepts in the Study of Complex Systems	12
1.2.1 Defining Characteristics	13
1.2.2 Basic Ingredients	14
1.2.3 Main Emergent Features	21
1.3 Self-organized Criticality	25
1.3.1 The Running Sandpile	26
1.3.2 Criticality in the SOC State	28
1.3.3 Memory in the SOC State	30
1.3.4 Transport in the SOC State	32
1.4 Overview of the First Part of This Book	34
Appendix 1: Fixed Points of a Dynamical System	35
Problems	36
References	37
2 Statistics	41
2.1 Introduction	41
2.2 The Probability Density Function	42
2.2.1 Definition	42
2.2.2 Cumulative Distribution Function	43
2.2.3 Survival Function	44
2.2.4 Characteristic Function	45
2.2.5 Expected Values	47
2.2.6 Moments	47
2.2.7 Cumulants	48

2.3	Significance of Specific Pdfs	51
2.3.1	Gaussian and Lévy Pdfs: Additive Processes	52
2.3.2	Log-Normal and Log-Stable Pdfs: Multiplicative Processes	57
2.3.3	Weibull, Gumbel and Frechet Pdfs: Extreme Value Pdfs	61
2.3.4	Exponential and Related Pdfs: Poisson Processes	64
2.4	Techniques for the Practical Estimation of Pdfs	70
2.4.1	Constant Bin Size Method	71
2.4.2	Constant Bin Content Method	73
2.4.3	Survival/Cumulative Distribution Function Method	78
2.5	Techniques to Compare Experimentally Obtained Pdfs with Analytical Forms	81
2.5.1	Maximum Likelihood Estimators	81
2.5.2	Pearson's Goodness-of-Fit Test	85
2.5.3	Minimum Chi-Square Parameter Estimation	89
2.6	Case Study: The Running Sandpile	90
2.7	Final Considerations	94
Appendix 1:	The Fourier Transform	94
Appendix 2:	Numerical Generation of Series with Prescribed Statistics	97
Problems	99	
References	101	
3	Scale Invariance	103
3.1	Introduction	103
3.2	Scale-Invariance in Space	106
3.2.1	Fractals	107
3.2.2	Multifractals	111
3.3	Scale-Invariance in Time	120
3.3.1	Self-Similar Time Random Processes	121
3.3.2	Propagator of a Random Process	122
3.3.3	Fractional Brownian Motion	130
3.3.4	Fractional Lévy Motion	133
3.3.5	Stationarity and Self-Similarity	135
3.3.6	Self-Similar Processes with Stationary Increments	138
3.3.7	Multifractal Time Random Processes	140
3.4	Techniques for the Practical Determination of Scale-Invariance	145
3.4.1	Analysis of Non-stationary Processes	146
3.4.2	Analysis of Stationary Processes	150
3.4.3	Multifractal Analysis	156
3.5	Case Study: The Running Sandpile	162
3.6	Final Considerations	167
Appendix 1:	Numerical Generation of Fractional Noises	167
Appendix 2:	Detrended Fluctuation Analysis	169
Appendix 3:	Multifractal Analysis via Wavelets	171
Problems	172	
References	173	

4	Memory	177
4.1	Introduction	177
4.2	Memory and Correlation	178
4.2.1	The Autocorrelation Function	178
4.2.2	The Power Spectrum	188
4.2.3	The Autodifference Function	193
4.3	Memory in Self-Similar Time Random Processes	195
4.3.1	Fractional Brownian Motion	195
4.3.2	Fractional Lévy Motion	196
4.4	Techniques for Detecting Memory in Stationary Time Series	198
4.4.1	Methods Based on the Autocorrelation Function	199
4.4.2	Methods Based on the Power Spectrum	200
4.4.3	Methods Based on the Autodifference	202
4.4.4	Hurst's Rescaled Range (R/S) Method	203
4.4.5	Waiting Time Statistics	212
4.5	Case Study: The Running Sandpile	213
4.6	Final Considerations	218
Problems	218	
References	219	
5	Fundamentals of Fractional Transport	221
5.1	Introduction	221
5.2	Diffusive Transport: Fundamentals	223
5.2.1	The Continuous-Time Random Walk	223
5.2.2	The Langevin Equation	230
5.3	Scale Invariant Formulations of Transport	234
5.3.1	Scale Invariant Continuous-Time Random Walks	234
5.3.2	The Fractional Langevin Equation	241
5.3.3	The Fractional Transport Equation	243
5.4	Techniques for the Characterization of Fractional Transport	247
5.4.1	Eulerian Methods	247
5.4.2	Lagrangian Methods	252
5.5	Case Study: The Running Sandpile	255
5.5.1	fTe for the Directed Running Sandpile	260
5.6	Final Considerations	261
Appendix 1:	The Laplace Transform	264
Appendix 2:	Riemann-Liouville Fractional Derivatives and Integrals	265
Appendix 3:	The Riesz-Feller Fractional Derivative	269
Appendix 4:	Discrete Approximations for Fractional Derivatives	270
Problems	273	
References	274	

Part II Complex Dynamics in Magnetized Plasmas

6 Laboratory Fusion Plasmas: Dynamics of Near-Marginal Turbulent Radial Transport	279
6.1 Introduction	279
6.2 Nuclear Fusion Processes	280
6.3 Primer on Magnetic Confinement Fusion	284
6.3.1 Tokamaks	285
6.3.2 Stellarators	287
6.3.3 Main Transport Processes in Toroidal MCF Plasmas	288
6.4 Are MCF Plasmas Complex Systems?	291
6.4.1 Tokamak Transport Phenomenology	292
6.4.2 Stellarator Confinement Phenomenology	295
6.4.3 Self-organized Criticality and Toroidal MCF Plasmas	296
6.5 Case Study: Analysis of Turbulent Fluctuations from the Edge of the W7-AS Stellarator	299
6.5.1 Statistics	300
6.5.2 Power Spectrum	303
6.5.3 R/S Analysis	304
6.5.4 Multifractal Analysis	306
6.6 Conclusions	308
References	309
7 Space Plasmas: Complex Dynamics of the Active Sun	313
7.1 Introduction	313
7.2 Our Own Star: The Sun	313
7.2.1 Structure of the Sun	314
7.2.2 The Active Magnetic Sun	317
7.3 Is Our Sun a Complex System?	319
7.3.1 The Tachocline: A Case of Self-Organization	320
7.3.2 Scale-Invariance of Solar Flare Data	321
7.3.3 Lu-Hamilton SOC Flaring Model	322
7.4 Case Study: Analysis of the SOHO-LASCO CME Database (1996–2016)	324
7.4.1 Waiting-Time Statistics	326
7.4.2 Linear Speed Analysis	326
7.4.3 Ejected Mass Analysis	329
7.4.4 Ejected Energy Analysis	333
7.5 Conclusions	335
References	335
8 Planetary Plasmas: Complex Dynamics in the Magnetosphere of the Earth	339
8.1 Introduction	339
8.2 The Magnetosphere of the Earth	340
8.2.1 The Geomagnetic Field	341

8.2.2 Structure of the Magnetosphere of the Earth	342
8.2.3 Dynamics of the Magnetosphere of the Earth	347
8.3 Is the Magnetosphere of the Earth a Complex System?	352
8.3.1 Chang's SOC Substorming Model	353
8.3.2 Evidence of Critical Dynamics in the Magnetotail	356
8.4 Case Study: Magnetospheric and Solar Wind Indices	356
8.4.1 Analysis of the D_{st} Index (1957–2008)	357
8.4.2 Analysis of the AE Index (1990–2008)	363
8.4.3 Analysis of the Scalar B in the Solar Wind (1963–2017)	369
8.4.4 Analysis of the Proton Density in the Solar Wind (1963–2017)	374
8.5 Conclusions	378
References	378
9 Laboratory Plasmas: Dynamics of Transport Across Sheared Flows	381
9.1 Introduction	381
9.2 Stable Sheared Flows	382
9.2.1 Differential Rotation and Magnetic Fields	383
9.2.2 Turbulence Suppression by Sheared Flows	384
9.2.3 Zonal Flows in Tokamaks	386
9.3 Non-diffusive Transport Across Sheared Flows	388
9.4 Case Study: Transport Across Self-Consistent Zonal Flows in Ion-Temperature-Gradient (ITG) Tokamak Turbulence	391
9.4.1 Radial Propagator Analysis	395
9.4.2 Other Considerations	397
9.5 Conclusions	399
References	399
Index	401