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^{*}Sections marked by an asterisk contain advanced material that may be omitted on a first reading.

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