

# Contents

<b>1</b>	<b>Introduction: What Is This Book About?</b>	<b>1</b>
1.1	Main Ideas . . . . .	1
1.2	Subject Matter . . . . .	2
References . . . . .		5
<b>2</b>	<b>Box of Tools</b> . . . . .	<b>7</b>
2.1	Wave Functions and Propagators . . . . .	7
2.1.1	System of Units . . . . .	7
2.1.2	Some Aspects of Quantum Electrodynamics . . . . .	8
2.1.3	The Lippmann–Schwinger Equation . . . . .	15
2.2	Two Scales of Momenta . . . . .	17
2.2.1	Differential Cross Sections for Bound and Free Electrons . . . . .	17
2.2.2	The Bethe Ridge . . . . .	19
2.2.3	Transfer of Large Momenta . . . . .	21
References . . . . .		24
<b>3</b>	<b>Perturbation Theory</b> . . . . .	<b>25</b>
3.1	Interaction of the Fast Electron and the Nucleus . . . . .	25
3.1.1	Lowest-Order Correction . . . . .	25
3.1.2	Wave Function of the Continuum State Electron at the Origin . . . . .	27
3.1.3	Inelastic Electron Scattering by Atoms . . . . .	29
3.1.4	Use and Misuse of Plane Waves . . . . .	33
3.2	Final State Interactions Between Electrons . . . . .	35
3.2.1	General Analysis . . . . .	35
3.2.2	Zero Order Terms . . . . .	36
3.2.3	First Order Terms . . . . .	38
3.2.4	Second-Order Terms . . . . .	43
3.2.5	Probabilities of the Exclusive Processes . . . . .	45

	Contents	Contents			
3.2.6	Probability of the Inclusive Process . . . . .	46	5.4	Second Order Processes II: Raman and Compton Scattering . . . . .	100
3.2.7	The Relativistic Case . . . . .	47	5.4.1	Raman Scattering . . . . .	100
References	. . . . .	49	5.4.2	Compton Scattering . . . . .	103
<b>4</b>	<b>Singularities of Amplitudes and Wave Functions . . . . .</b>	<b>51</b>	5.4.3	Total Cross Section of Photon Scattering . . . . .	110
4.1	General Features of the Reactions $2 \rightarrow 3$ . . . . .	51	5.5	Expansion in Powers of $1/Z$ . . . . .	111
4.1.1	Amplitudes Outside the Bethe Ridge . . . . .	51	5.5.1	Ground-State Energies of Heliumlike Ions . . . . .	111
4.1.2	Triangle Diagrams in the Case of the Coulomb Field. . . . .	55	5.5.2	Photoionization of Helium Near the Threshold. . . . .	112
4.1.3	Triangle Diagrams in the Case of Short-Range Forces . . . . .	56	References	. . . . .	114
4.1.4	Amplitudes on the Bethe Ridge in the Presence of Singularities . . . . .	57	<b>6</b>	<b>The Coulomb Field. Relativistic Case . . . . .</b>	<b>115</b>
4.1.5	Contribution of Triangle Diagrams to Differential Cross Sections. . . . .	60	6.1	Wave Functions . . . . .	115
4.2	Fast Secondary Electrons . . . . .	63	6.1.1	Wave Functions with Fixed Angular Momentum . . . . .	115
4.2.1	Angular Correlations and Energy Distributions . . . . .	63	6.1.2	The Continuum Wave Function in the Ultrarelativistic Limit. . . . .	117
4.2.2	Internal Energy Loss . . . . .	66	6.1.3	Furry–Sommerfeld–Maue Approximation . . . . .	119
4.3	Kato Cusp Conditions . . . . .	68	6.2	The $\alpha Z$ Dependence of Electron Functions . . . . .	123
4.3.1	Electron–Nucleus Coalescence Point. . . . .	69	6.2.1	Power Series for Wave Functions. . . . .	123
4.3.2	Electron–Electron Coalescence Point . . . . .	70	6.2.2	Relativistic Functions in Terms of FSM Functions . . . . .	124
4.3.3	A Wave Function Based on the Kato Cusp Condition . . . . .	71	6.3	Photoeffect . . . . .	127
4.4	Wave Functions of Helium and Heliumlike Ions . . . . .	73	6.3.1	General Remarks . . . . .	127
4.4.1	Three-Particle Coalescence Point . . . . .	73	6.3.2	Threshold Ionization of Heavy Ions . . . . .	128
4.4.2	Account of Analytical Properties in Approximate Wave Functions. . . . .	74	6.3.3	The $\alpha Z$ Dependence of Amplitude . . . . .	130
4.4.3	Approximate Wave Functions on Coalescence Lines . . . . .	75	6.3.4	Calculations in the Lowest Order in $\alpha Z$ . . . . .	131
References	. . . . .	78	6.3.5	Inclusion of Higher-Order Terms . . . . .	133
<b>5</b>	<b>The Coulomb Field. Nonrelativistic Case . . . . .</b>	<b>81</b>	6.3.6	Ultrarelativistic Case. . . . .	136
5.1	Wave Functions and Propagator. . . . .	81	6.4	Elastic Scattering of the High Energy Photons on Atoms . . . . .	138
5.1.1	General Remarks . . . . .	81	6.4.1	Channels of the Process . . . . .	138
5.1.2	Technique of Calculations . . . . .	82	6.4.2	Rayleigh Scattering . . . . .	138
5.1.3	Wave Functions of the Bound States . . . . .	83	6.4.3	Delbrück Scattering . . . . .	142
5.1.4	Wave Functions of the Continuum States . . . . .	85	6.4.4	Scattering on the Nucleus . . . . .	143
5.1.5	Examples of Applications . . . . .	87	6.4.5	Isolation of Partial Contributions . . . . .	144
5.1.6	Green Function . . . . .	88	6.5	Compton Scattering . . . . .	145
5.2	Photoeffect . . . . .	92	6.5.1	General Relations. . . . .	145
5.3	Second Order Processes I . . . . .	94	6.5.2	Lowest Order Calculations on the Bethe Ridge . . . . .	147
5.3.1	General Analysis . . . . .	94	6.5.3	Inclusion of Higher Order Terms on the Bethe Ridge . . . . .	151
5.3.2	Amplitude of Rayleigh Scattering. . . . .	95	6.5.4	Outside the Bethe Ridge . . . . .	153
5.3.3	Cross Section of the Rayleigh Scattering. . . . .	98	References	. . . . .	155
<b>7</b>	<b>Photoionization of Atoms . . . . .</b>	<b>157</b>	<b>7</b>	<b>Photoionization of Atoms . . . . .</b>	<b>157</b>
7.1	High-Energy Nonrelativistic Asymptotics . . . . .	157	7.1.1	Ionization of $s$ States . . . . .	158
7.1.1	Ionization of $s$ States . . . . .	158	7.1.2	Ionization of States with $\ell \neq 0$ . . . . .	160
7.1.2	Ionization of States with $\ell \neq 0$ . . . . .	160	7.1.3	Possibility of Asymptotic Analysis . . . . .	163
7.1.3	Possibility of Asymptotic Analysis . . . . .	163	7.1.4	Preasymptotic Behavior of the Cross Sections . . . . .	166

<b>7.2</b>	Forms of Electromagnetic Interactions . . . . .	167
7.2.1	Forms of Interaction and Gauge Invariance . . . . .	167
7.2.2	Thomas–Reiche–Kuhn Sum Rule . . . . .	173
7.2.3	Amplitude of Photoionization in Length Form . . . . .	175
<b>7.3</b>	Amplitude of Rayleigh Scattering in Length Form . . . . .	176
<b>7.3</b>	Relativistic Case . . . . .	177
7.3.1	The Lowest Order $\alpha Z$ Terms . . . . .	177
7.3.2	Far Away from the Threshold . . . . .	179
7.3.3	In the Vicinity of the Threshold . . . . .	183
<b>7.4</b>	Photoionization Beyond the Independent Particle Approximation . . . . .	184
7.4.1	Correlations in the L Shell . . . . .	184
7.4.2	Random Phase Approximation with Exchange . . . . .	190
7.4.3	Correlations in the Higher Subshells . . . . .	193
7.4.4	Intershell Correlations . . . . .	195
7.4.5	Nonrelativistic High-Energy Asymptotics . . . . .	197
7.4.6	Peculiarities of the Relativistic Case . . . . .	199
	References . . . . .	200
<b>8</b>	<b>Ionization and Excitation by Photon Impact at Higher Energies</b> . . . . .	203
8.1	Compton Scattering . . . . .	203
8.1.1	Interpretation of the Seagull Term . . . . .	203
8.1.2	Distribution of the Scattered Photons . . . . .	206
8.1.3	Distribution of Ejected Electrons . . . . .	210
8.1.4	Radiation of Soft Photons . . . . .	211
8.2	Ionization Accompanied by Creation of Pairs . . . . .	214
8.2.1	Vacuum Assistance Mechanism . . . . .	214
8.2.2	Energy Distribution of the Ejected Electrons . . . . .	216
8.2.3	Total Cross Section . . . . .	221
8.3	Excitation Accompanied by the Pair Creation . . . . .	223
8.3.1	Mechanisms of the Excitation Processes . . . . .	223
8.3.2	Competition of the Contributions . . . . .	226
	References . . . . .	229
<b>9</b>	<b>Double Photoionization and Related Processes</b> . . . . .	231
9.1	The General Picture . . . . .	231
9.1.1	Objects of Investigation . . . . .	231
9.1.2	Mechanisms of the Process . . . . .	232
9.2	Double Ionization in the Dipole Approximation . . . . .	235
9.2.1	Nonrelativistic High-Energy Asymptotics for Helium . . . . .	235
9.2.2	Nuclear Charge Dependence of Asymptotics for Heliumlike Ions . . . . .	241
9.2.3	Double Photoionization at Intermediate Energies . . . . .	246

9.2.4	Photoionization Followed by Excitation: Intermediate Energies . . . . .	254
9.2.5	Two Fast Photoelectrons . . . . .	258
<b>9.3</b>	Quasifree Mechanism . . . . .	263
9.3.1	The Amplitude . . . . .	263
9.3.2	Evaluation of the Shape of the Spectrum Curve . . . . .	265
9.3.3	High Energy Behavior of Ionization Cross Section Ratios . . . . .	269
9.3.4	Distributions in Recoil Momenta . . . . .	272
<b>9.4</b>	Ejection of Relativistic Electrons . . . . .	276
9.4.1	Distribution of Photoelectrons . . . . .	276
9.4.2	Energy Dependence of the Cross Section . . . . .	282
<b>9.5</b>	Two-Electron Capture with Emission of a Single Photon . . . . .	284
9.5.1	Experiment and Theory . . . . .	284
9.5.2	The High-Energy Case . . . . .	286
	References . . . . .	289
<b>10</b>	<b>Photoionization of Endohedral Atoms</b> . . . . .	291
10.1	Photoionization of Fullerenes . . . . .	291
10.1.1	Fullerenes . . . . .	291
10.1.2	Photodetachment of $C_{60}$ . . . . .	295
10.1.3	Asymptotics of the Photoionization Cross Section . . . . .	300
10.2	Photoionization of Caged Atoms . . . . .	302
10.2.1	Wave Functions of Caged Atoms . . . . .	302
10.2.2	Polarization of the Fullerene Shell . . . . .	304
10.2.3	Energy Dependence of the Photoionization Cross Section . . . . .	308
10.3	Absorption of Photoelectrons by the Fullerene Shell . . . . .	311
10.3.1	Photoelectron Interaction with the Fullerene Shell . . . . .	311
10.3.2	High-Energy Limit . . . . .	316
10.3.3	Energy Dependence of the Probability of Excitation of the Fullerene Shell . . . . .	319
	References . . . . .	320
<b>11</b>	<b>Annihilation of Positrons with Atomic Electrons</b> . . . . .	323
11.1	Two-Photon Annihilation . . . . .	323
11.1.1	On the Bethe Ridge: Fast Positrons . . . . .	323
11.1.2	On the Bethe Ridge: Slow Positrons . . . . .	327
11.1.3	Photon Distribution Outside the Bethe Ridge . . . . .	329
11.2	Annihilation with Radiation of One Photon . . . . .	331
11.2.1	Single-Quantum Annihilation . . . . .	331
11.2.2	Annihilation Followed by Ionization . . . . .	332

11.3 Annihilation Without Radiation . . . . .	336
11.3.1 Annihilation with Ionization . . . . .	336
11.3.2 Annihilation with Creation of $\mu^+ \mu^-$ Pairs. . . . .	340
References . . . . .	344
<b>12 Nuclear Transitions and the Electron Shell . . . . .</b>	<b>345</b>
12.1 Role of Atomic Electrons in Nuclear Beta Decay. . . . .	345
12.1.1 Amplitude of Nuclear Beta Decay . . . . .	345
12.1.2 Neutrino Mass Measurements . . . . .	347
12.1.3 A Tale of a Heavy Neutrino . . . . .	353
12.1.4 Creation of Vacancies in the Electron K Shell in $\beta$ Decay. . . . .	356
12.1.5 Creation of Vacancies in the Electron K Shell in $\beta^+$ Decay. . . . .	360
12.2 Interactions of Gamma Quanta with the Electron Shell . . . . .	363
12.2.1 Amplitude for Electromagnetic Transition of the Nucleus . . . . .	363
12.2.2 Internal Nuclear Conversion . . . . .	364
12.2.3 Two-Electron Processes in the Electron Shell . . . . .	367
12.2.4 Excitation of Nuclear Levels by Electronic Transitions . . . . .	369
12.3 Electron Shell in Alpha Decays . . . . .	373
12.3.1 Transitions in Internal Shells . . . . .	373
12.3.2 Influence of the Electron Shell on the Probability of Alpha Decay . . . . .	376
References . . . . .	381
<b>Index . . . . .</b>	<b>383</b>