## Contents

1	Intro	ducing Reflection	1
	1.1	The Electromagnetic s Wave	1
	1.2	The Electromagnetic p Wave	6
	1.3	Particle Waves	12
	1.4	Acoustic Waves	16
	1.5	Scattering and Reflection	20
	1.6	A Look Ahead	24
	Refer	rences	39
2	Exac	t Results	41
	2.1	Comparison Identities, and Conservation	
		and Reciprocity Laws	41
	2.2	General Expressions for $r_s$ and $r_p$	46
	2.3	Reflection at Grazing Incidence, and the Existence	
		of a Principal Angle	52
	2.4	Reflection by a Homogeneous Layer	55
	2.5	The Tanh, Exp and Rayleigh Profiles	61
	Refer	rences	72
3	Refle	ection of Long Waves	75
	3.1	Integral Equation and Perturbation Theory for the s Wave	75
	3.2	The s Wave to Second Order in the Interface Thickness	79
	3.3	Integral Invariants	81
	3.4	$ r_p ^2$ and $r_p/r_s$ to Second Order	84
	3.5	Reflection by a Thin Film Between Like Media	88
	3.6	Six Profiles and Their Integral Invariants	90
	Refe	rences	93
4	Varia	ational Theory	95
	4.1	A Variational Expression for the Reflection Amplitude	95
	4.2	Variational Estimate for $r_s$ in the Long Wave Case	98

х

	4.3	Exact, Perturbation and Variational Results	0.0	
		for the sech <sup>2</sup> Profile	99	
	4.4	Variational Theory for the p Wave	103	
	4.5	Reflection by a Layer Between Like Media	106	
	4.6	The Hulthén-Kohn Variational Method Applied		
		to Reflection.	110	
	4.7	Variational Estimates in the Short Wave Case	112	
	Refere	ences	114	
5	Equa	tions for the Reflection Amplitudes	115	
	5.1	A First Order Non-linear Equation for an s Wave Reflection		
		Coefficient	115	
	5.2	An Example: Reflection by the Linear Profile	117	
	5.3	Differential Equation for a <i>p</i> Wave Reflection Coefficient	120	
	5.4	Upper Bounds on $R_s$ and on $R_p$	122	
	5.5	Long Wave Expansions	124	
	5.6	Differential Equations for the Reflection Amplitudes	128	
	5.7	Weak Reflection: The Rayleigh Approximation	130	
	5.8	Iteration of the Integral Equation for $r$	131	
	Refere	ences	133	
6	Refle	ction of Short Waves	135	
	6.1	Short Wave Limiting Forms for Some Solvable Profiles	135	
	6.2	Approximate High-Frequency Waveforms	139	
	6.3	Profiles of Finite Extent with Discontinuities in Slope		
		at the Endpoints	142	
	6.4	Reflection Amplitude Estimates from		
		a Comparison Identity	145	
	6.5	Perturbation Theory for Short Waves.	150	
	6.6	Short Wave Results for $r_p$ and $r_p/r_s$	152	
	6.7	A Single Turning Point: Total Reflection	159	
	6.8	Two Turning Points, and Tunneling	166	
	Refer	ences	172	
7	Simp	le Anisotropy	175	
	7.1	Anisotropy with Azimuthal Symmetry	175	
	7.2	Ellipsometry of a Thin Film on an Isotropic Substrate	179	
	7.3	Thin Film on an Anisotropic Substrate.	182	
	7.4	General Results for Anisotropic Stratifications		
		with Azimuthal Symmetry	184	
	7.5	Differential Equations for the Reflection Amplitudes	185	
	7.6	Reflection from the Ionosphere	187	
	Refer	ences	189	

8	Unia	xial Anisotropy	191
	8.1	Propagation Within Homogeneous Anisotropic Media	191
	8.2	Dielectric Tensor and Normal Modes in Uniaxial Crystals	193
	8.3	Uniaxial Crystal Reflection and Transmission Amplitudes	196
	8.4	Bounds and Zeros of the Reflection Amplitudes,	
		the Polarizing Angle	199
	8.5	External Reflection from an Immersed Crystal	201
	8.6	Normal-Incidence Reflection and Transmission	202
	8.7	Normal Incidence on a Uniaxial Plate	205
	8.8	Isotropic Layer on a Uniaxial Substrate	209
	8.9	Optical Properties of a Uniaxial Layer	211
	Refer	ences	212
Q	Fllin	sometry	215
,	0.1	Polarizer_Sample_Analyser	215
	0.2	Polarizer_Compensator_Sample_Analyser	217
	0.3	Polarizer_Sample_Compensator_Analyser	218
	9.5	Polarizer Modulator Sample_Analyser	210
	9. <del>4</del> 0.5	Polarizer Sample Modulator Analyser	221
	9.5	Foranzer-Sample-Modulator-Analyser	221
	9.0	Transmission Ellincometry	221
	9.7	0.7.1 Polorizor Sample Applyer	222
		9.7.1 Foldizer-Sample-Analyser	223
		9.7.2 Folarizer Compensator Analyser	223
		9.7.5 Foranzei-Sample-Compensator-Analyser	224
		9.7.4 Italishiission Empsoneury with a Folanzation Modulator	225
	0.0	Poflection and Transmission Ellipsometry	443
	9.0	of a Hamogeneous Laver	225
	0.0	Deflection Ellingemetry of Uniovial Crystals	225
	9.9 D.f.	Reflection Empsometry of Omaxia Crystals	220
	Refer	ences	250
10	Abso	rption	233
	10.1	Fresnel Reflection Formulae for an Absorbing Medium	234
	10.2	General Results for Reflection by Absorbing Media	240
	10.3	Dielectric Layer on an Absorbing Substrate	241
	10.4	Absorbing Film on a Non-absorbing Substrate	242
	10.5	Thin Inhomogeneous Absorbing Films	245
	10.6	Attenuated Total Reflection, Surface Waves	249
	10.7	Attenuated Total Reflection: Second Example	256
	10.8	Reflection by a Diffuse Absorbing Interface:	
		The Tanh Profile	259
	10.9	Zero Reflection from Dielectric Layer	
		on Absorbing Substrate	262
	Refe	rences	262

Contents

11	Inver	se Problems	265
	11.1	Reflection at a Sharp Boundary	266
	11.2	Homogeneous Film Between Like Media	269
	11.3	Inversion of Transmission Ellipsometric Data	
		for a Homogeneous Nonabsorbing Layer	271
	11.4	Inversion of Reflection Ellipsometric Data	
		for a Homogeneous Nonabsorbing Layer	272
	11.5	Synthesis of a Profile from $r$ as a Function of Wavenumber	273
	11.6	Inversion of the Rayleigh Approximation	276
	11.7	Principal Angle of an Absorber.	278
	Refer	ences	279
12	Matr	ix and Numerical Methods	281
	12.1 Matrices Relating the Coefficients of Linearly Independent		
		Solutions	281
	12.2	Matrices Relating Fields and Their Derivatives	285
	12.3	Multilayer Dielectric Mirrors at Normal Incidence.	290
	12.4	Reflection of Long Waves	293
	12.5	Absorbing Stratified Media: Some General Results	295
	12.6	High Transparency of an Absorbing Film	
		in a Frustrated Total Reflection Configuration	298
	12.7	Comparison of Numerical Approaches	300
	12.8	Numerical Methods Based on the Layer Matrices	301
	12.9	Variable Step Size, Profile Truncation, Total Reflection	
		and Tunneling, Absorption, and Calculation	
		of Wavefunctions	306
	Refer	ences	309
13	Perio	dically Stratified Media	311
	13.1	Electromagnetic Waves in Stratified Media.	312
	13.2	Periodic Structures, Multilayer Dielectric Mirrors	317
	13.3	Omnidirectional Reflection by Multilayer Dielectric Mirrors	323
		13.3.1 Band Edges at Oblique Incidence	
		for a General Stack	325
		13.3.2 Refractive Indices for Which Omnidirectional	
		Reflection Exists	327
	13.4	Form Birefringence	329
	13.5	Absorbing Periodically Stratified Media	332
		13.5.1 Reflection of <i>s</i> -Polarized Plane Waves	333
		13.5.2 Reflection of <i>p</i> -Polarized Plane Waves	335
		13.5.3 Application to an Absorbing Quarter-Wave Stack	337
	Refe	rences	338
			2.41
1/	Dave	The AP STRUCTURAD SUPTODAS	4/11
14	Roug	Baflection from Rough Surfaces: The Rayleigh Criterion	341
14	<b>Roug</b>	Reflection from Rough Surfaces: The Rayleigh Criterion	341 342 343

	14.3 14 4	Scattering of Light by Liquid Surfaces	349
	17.7	by Rough Surfaces	353
	14.5	Absorbing and Rough Surfaces that Are Wet	356
	14.6	Coherent Backscattering.	358
	Refere	ences	359
15	Partic	ele Waves	363
	15.1	General Results.	363
	15.2	Some Exactly Solvable Profiles	367
	15.3	Perturbation and Variational Theories	374
	15.4	Long Waves, Integral Invariants	376
	15.5	Riccati-Type Equations; the Rayleigh Approximation	378
	15.6	Reflection of Short Waves	380
	15.7	Absorption, the Optical Potential.	382
	15.8	Inversion of a Model Reflection Amplitude	385
	15.9	Time Delay in the Reflection of Wavepackets	387
	Refere	ences	390
16	Neutr	on and X-ray Reflection	391
10	16.1	Common Features of X-ray and Neutron Optics	392
	16.2	Reflection Near the Critical Angle.	393
	16.3	Reflection by Profiles Without Discontinuities	397
	16.4	Reflection by Profiles with Discontinuities	400
	16.5	Total Reflection: Extraction of the Phase in Llovd's	
		Mirror Experiments	405
	16.6	Reflection of Neutrons by Periodic Stratifications	410
	16.7	Neutron Reflection by Magnetic Materials	414
	Refer	ences	416
17	A	tia Waxaa	410
17	Acou	Concerned Delations for Stratified Modia	419
	17.1	17.1.1 General Pacults for the Deflection	417
		and Transmission Amplitudes	420
	17.2	Matrix Methode	425
	17.2	Low Frequency Reflection and Transmission	431
	17.3	High-Frequency Limiting Forms	434
	17.4	Exact Solutions for the <i>arn_lin</i> and <i>arn_arn</i> Stratifications	439
	17.5	An Upper Bound on the Acoustic Reflectivity	441
	17.0	Profiles with Discontinuities in Density or Sound Speed	444
	Δnne	ndiv: Universal Properties of Acoustic Pulses and Reams	448
	Refer	ences	451
	Reich	cnees	
18	Chira	al Isotropic Media	453
	18.1	Constitutive Relations	454
	18.2	Reflection and Transmission Amplitudes, Conservation Laws	456

		18.2.1 Differential Reflectance, Ellipsometry	460
	18.3	Wave Propagation in Chiral Media	461
		18.3.1 Eigenstates of Curl	463
		18.3.2 Boundary Conditions	464
	18.4	Reflection from an Achiral–Chiral Interface	465
		18.4.1 Wavefunctions	466
		18.4.2 Reflection and Transmission Amplitudes	466
		18.4.3 The Angles $\theta_B$ , $\theta_{pp}$ , and $\theta_{pol}$	470
	18.5	Optical Properties of a Chiral Layer	471
		18.5.1 Normal Incidence	472
		18.5.2 Optical Properties Near the Critical Angles.	474
	Refer	ences	475
19	Pulse	s and Wavepackets	477
	19.1	Reflection of Nearly Monochromatic Pulses:	
		The Time Delay	477
	19.2	Nonreflection of Wavepackets by a Subset of the sech <sup>2</sup>	
		Potentials	481
		19.2.1 Construction of Reflectionless Wavepackets	482
	19.3	Exact Solutions of Total and Partial Reflection	
		of Wavepackets	485
	Appe	ndix: Universal Properties of Electromagnetic Pulses	489
	Refer	ences	497
20	Finite	e Beams	499
	20.1	Universal Properties of Scalar and of Electromagnetic	
		Beams	499
		20.1.1 Bateman Integral Solution of the Wave Equation	501
		20.1.2 Conservation Laws and Beam Invariants	502
		20.1.3 Non-existence Theorems	504
		20.1.4 Focal Plane Zeros	505
	20.2	Reflection of Beams: The Lateral Beam Shift	507
	20.3	Reflection of Gaussian Beams	511
		20.3.1 Reflection at a Potential Spike (Delta Function)	513
		20.3.2 Reflection at a Sharp Boundary Between	
		Two Media	514
	Appe	ndix 1: Total Internal Reflection: The $r_s$ , $r_p$ Phases	
		and Their Difference	515
	Appe	ndix 2: Polarization of Electromagnetic Beams	521
	Refer	ences	526
	1.	D. A. Han and Thomas incide Formulas	520
Ap	pendix	: Kellection and Transmission Formulae	329
Ind	lex		535