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# Gaussian Beam Propagation in Inhomogeneous Nonlinear Media. Description in Ordinary Differential Equations by Complex Geometrical Optics

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# Single-Particle Cryo-Electron Microscopy (Cryo-EM): Progress, Challenges, and Perspectives for Further Improvement

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# Morphological Amoebas and Partial Differential Equations

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