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## INTRODUCTION TO QUANTUM INFORMATION AND COMPUTATION FOR CHEMISTRY

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# BACK TO THE FUTURE: A ROADMAP FOR QUANTUM SIMULATION FROM VINTAGE QUANTUM CHEMISTRY

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- II. Quantum Computing
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# INTRODUCTION TO QUANTUM ALGORITHMS FOR PHYSICS AND CHEMISTRY

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# QUANTUM COMPUTING APPROACH TO NONRELATIVISTIC AND RELATIVISTIC MOLECULAR ENERGY CALCULATIONS

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# DENSITY FUNCTIONAL THEORY AND QUANTUM COMPUTATION

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# **ANALYTIC TIME EVOLUTION, RANDOM PHASE APPROXIMATION, AND GREEN FUNCTIONS FOR MATRIX PRODUCT STATES**

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- II. Stationary States
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# FEW-QUBIT MAGNETIC RESONANCE QUANTUM INFORMATION PROCESSORS: SIMULATING CHEMISTRY AND PHYSICS

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# PHOTONIC TOOLBOX FOR QUANTUM SIMULATION

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# FUNCTIONAL SUBSYSTEMS AND STRONG CORRELATION IN PHOTOSYNTHETIC LIGHT HARVESTING

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- I. Introduction
- II. Effect of Strong Electron Correlation
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## I. INTRODUCTION

Nature harvests solar energy with a remarkably high *quantum efficiency*, the percentage of charge carriers created by photons. Recent spectroscopic experiments [1–3] and theoretical models [4–13], provide strong evidence that efficient light harvesting in nature occurs by a quantum mechanism involving sustained electronic coherence [14] and entanglement [15, 16] between chromophores. Quantum coherence in the Fenna–Matthews–Olson (FMO) antennae complex of green

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# VIBRATIONAL ENERGY TRANSFER THROUGH MOLECULAR CHAINS: AN APPROACH TOWARD SCALABLE INFORMATION PROCESSING

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- II. Fundamentals of Quantum Dynamics and Coherent Control
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## I. INTRODUCTION

Quantum information processing is a rapidly developing field and has entered different areas in physics and chemistry. The first principal ideas came from the quantum optics community and considerable success was reported with cavity

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# ULTRACOLD MOLECULES: THEIR FORMATION AND APPLICATION TO QUANTUM COMPUTING

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- II. Ultracold Molecule Formation
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# DYNAMICS OF ENTANGLEMENT IN ONE- AND TWO-DIMENSIONAL SPIN SYSTEMS

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# FROM TOPOLOGICAL QUANTUM FIELD THEORY TO TOPOLOGICAL MATERIALS

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- II. Topological Quantum Field Theory
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#### D. Effective Hamiltonians on Edges and Vortices

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References

# TENSOR NETWORKS FOR ENTANGLEMENT EVOLUTION

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I. Introduction  
II. Penrose Graphical Notation and Map-State Duality  
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