# Contents


*Preface* *page* ix
*Constants* xiii
*Notation* xv

**1 Newton's gravitational theory** 1
1.1 The law of universal gravitation 1
1.2 Tests of the inverse-square law 4
1.3 Gravitational potential 11
1.4 Gravitational multipoles; quadrupole moment of the Sun 13
1.5 Inertial and gravitational mass 17
1.6 Tests of equality of gravitational and inertial mass 19
1.7 Tidal forces 29
1.8 Tidal field as a local measure of gravitation 34
Problems 42
References 45

**2 The formalism of special relativity** 47
2.1 The spacetime of special relativity 48
2.2 Tensors in spacetime 55
2.3 Tensor fields 62
2.4 Energy-momentum tensor 64
2.5 Relativistic electrodynamics 71
2.6 Differential forms and exterior calculus 76
Problems 87
References 94

**3 The linear approximation** 95
3.1 The example of electromagnetism 95
3.2 Linear field equations for gravitation 101
3.3 Variational principle and equation of motion 106
3.4 Nonrelativistic limit and Newton's theory 112
3.5 Geometric interpretation; curved spacetime 117
Problems 123
References 126

**4 Applications of the linear approximation** 127
4.1 Field of a spherical mass 127
4.2 Gravitational time dilation 130
4.3 Deflection of light 138
4.4 Time delay of light 142
4.5 Gravitational lenses 149
4.6 Optics of gravitational lenses 159
4.7 Field of a rotating mass; Lense-Thirring effect 164
Problems 170
References 180

**5 Gravitational waves** 182
5.1 Plane waves 182
5.2 Interaction of particles with a gravitational wave 187
5.3 Emission of gravitational radiation 191
5.4 Emission by a vibrating quadrupole 196
5.5 Emission by a rotating quadrupole 199
5.6 Emission of bursts of gravitational radiation 204
5.7 Detectors of gravitational radiation 208
Problems 215
References 220

**6 Riemannian geometry** 221
6.1 General coordinates and tensors 223
6.2 Parallel transport; covariant derivative 226
6.3 Geodesic equation 232
6.4 Metric tensor 236
6.5 Riemann curvature tensor 243
6.6 Geodesic deviation and tidal forces; Fermi-Walker transport 252
6.7 Differential forms in curved spacetime 257
6.8 Isometries of spacetime; Killing vectors 262
Problems 268
References 274

**7 Einstein's gravitational theory** 275
7.1 General covariance and invariance; gauge transformations 276
7.2 Einstein's field equation 284
7.3 Another approach to Einstein's equation; cosmological term 288
7.4 Schwarzschild solution and Birkhoff theorem 293
7.5 Motion of planets; perihelion precession 299
7.6 Propagation of light; gravitational redshift 305
7.7 Geodetic precession 309
Problems 317
References 322

**8 Black holes and gravitational collapse** 324
8.1 Singularities and pseudosingularities 325
8.2 The black hole and its horizon 329
8.3 Maximal Schwarzschild geometry 335
8.4 Kerr solution and Reissner-Nordstrøm solution 343
8.5 Horizons and singularities of the rotating black hole 349
8.6 Maximal Kerr geometry 356
8.7 Black-hole thermodynamics; Hawking process 360
8.8 Gravitational collapse and formation of black holes 367
8.9 In search of black holes 375
Problems 381
References 387

**9 Cosmology** 389
9.1 Large-scale structure of the universe 390
9.2 Cosmic distances 392
9.3 Expansion of the universe; Hubble's law 394
9.4 Age of the universe 401
9.5 Cosmic background radiation 404
9.6 Mass density; dark mass 408
9.7 Comoving coordinates; Robertson-Walker geometry 411
9.8 Friedmann models ($\rho \neq 0$, $\Lambda = 0$) 418
9.9 Empty Lemaître models ($\rho = 0$, $\Lambda \neq 0$) 424
9.10 Friedmann-Lemaître models ($\rho \neq 0$, $\Lambda \neq 0$) 426
9.11 Propagation of light; particle horizon 428
9.12 Comparison of theory and observation 434
Problems 437
References 442

**10 The early universe** 444
10.1 Temperature of the early universe 445
10.2 Nucleosynthesis; abundance of primordial helium 451
10.3 Density perturbations; Jeans mass 456
10.4 Inflationary model 462
Problems 473
References 476

**Appendix: Variational principle and energy-momentum tensor** 477
A.1 Lagrange equations for a system of particles 477
A.2 Lagrange equations for fields 479
A.3 Energy-momentum tensor 482
A.4 Variational principle for Einstein's equations 486
A.5 Flux theorem and its implications for gravitational and inertial mass 491
References 496

*Answers to even-numbered problems* 497
*Index* 503