

Detailed Contents

<i>Preface</i>	<i>page xv</i>
1 The Finite Element Method: Introductory Remarks	1
The Mathematical Approach: A Variational Interpretation	2
Continuum Problems	3
Terminology and Preliminary Considerations: Types of Nodes	4
Degrees of Freedom	4
Interpolation Functions: Polynomials	6
One Independent Variable	6
Two Independent Variables	7
Three Independent Variables	7
Deriving Interpolation Functions	8
Natural Coordinates	10
Natural Coordinates in One Dimension	11
Natural Coordinates in Two Dimensions	12
Natural Coordinates in Three Dimensions	14
Curve-Sided Isoparametric Elements	16
Coordinate Transformation	17
Evaluation of Elemental Matrices	20
References	22
2 Some Methods for Solving Continuum Problems	23
Overview	23
The Ritz Method	24
Example: The Ritz Method	24
The Finite Element Method: Relation to the Ritz Method	26
3 Variational Approach	27
Example of Piecewise Approximation	28
Elemental Equations from a Variational Principle	30

4 **Requirements for the Interpolation Functions** 33

5 **Heat Transfer Applications** 35

 Variational Approach 35

 Example 37

 Approximation of Integrals 38

 One-Dimensional Steady-State Problems 39

 Finite Element Formulation 40

6 **One-Dimensional Steady-State Problems** 42

 Variational Statement 42

 Finite Element Formulation 44

 Numerical Results 48

 Problems 49

7 **The Two-Dimensional Heat-Conduction Problem** 51

 Variational Statement 51

 Finite Element Formulation 52

 Numerical Solution 59

 Numerical Results 63

8 **Three-Dimensional Heat-Conduction Applications with Convection and Internal Heat Absorption** 66

 The Problem of Cooling a Radial Turbine Rotor: Overview 66

 Governing Equations 67

 Finite Element Variational Formulation 68

 Euler’s Theorem of Variational Calculus 68

 Derivation of the Variational Statement 69

 Discretization of the Continuum 71

 Evaluation of $dI_k/d\{T\}$ 73

 Evaluation of $dI_g/d\{T\}$ 78

 Evaluation of $dI_h/d\{T\}$ 79

 Evaluation of $dI_q/d\{T\}$ 82

 The Final Set of Equations 83

 Turbine Rotor Configuration and Cooling Techniques 83

 Determination of the Hot Turbine Boundary Conditions 84

 Rotor Blade 84

 Rotor Disk Backside 85

 Rotor Hub 85

 Cooled Turbine Rotor Calculations 85

 Rotor Disk Cooling 86

 Blade Cooling through a Slot 86

 Blade Cooling through Radially Drilled Holes 86

 Numerical Results 87

 Remark 93

Problems	93
References	94
9 One-Dimensional Transient Problems	95
Variational Statement	95
Finite Element Formulation	96
Numerical Solutions	101
Euler's Method	101
Crank-Nicolson Method	103
Purely Implicit Method	103
References	105
10 Fluid Mechanics Finite Element Applications	106
Introduction	106
Inviscid Incompressible Flows	107
Problem Statement	107
Velocity Potential and Stream-Function Formulations	109
Flow around Multiple Bodies by Superposition	111
References	113
11 Use of Nodeless Degrees of Freedom	114
Overview	114
Flow-Governing Equations	120
Boundary Conditions	122
Flow Inlet Station ($B - C$)	122
Flow Exit Station ($D - A$)	122
Periodic Boundaries ($A - B$ and $D - C$)	123
Domain-Splitting Boundaries ($E - F$ and $G - H$)	123
Airfoil Suction and Pressure Surfaces	123
Finite Element Analysis	124
Galerkin's Weighted-Residual Approach	127
Applications	128
Flow Analysis in a Rectilinear-Airfoil Cascade	128
Field Discretization Model	128
Computational Results and Accuracy Assessment	128
Periodicity Conditions in Radial Cascades	130
Flow Investigation in a Radial-Turbine Scroll	130
Finite Element Analysis	132
Introduction of a Velocity Potential Discontinuity	134
Computational Results	134
Proposed Analysis Upgrades	135
Domains with High Degrees of Multiconnectivity	135
Axial-Flow Stator with a Spanwise Circulation	
Variation	138
Problems	138
References	147

12 Finite Element Analysis in Curvilinear Coordinate	149
Introduction	149
Analysis Guidelines and Limitations	153
Flow-Governing Equations	154
Continuity Equation	154
Through-Flow Momentum Equation	154
Tangential Momentum Equation	154
Boundary Conditions	156
Finite Element Formulation	158
Continuity Equation	160
Through-Flow Momentum Equation	160
Tangential Momentum Equation	160
Iterative Solution Procedure	161
Application Examples	162
Example 1: Second-Stage Stator of a Gas Turbine	164
Example 2: Low-Aspect-Ratio Turbine Stator	165
Proposed Analysis Upgrades	167
Adaptation to a Rotating-Blade Cascade	167
Inclusion of the Flow Turbulence Aspect	169
Problems	169
References	174
13 Finite Element Modeling of Flow in Annular Axisymmetric Passages	176
Introduction	176
Analysis	177
Flow-Governing Equations	177
Turbulence Closure	178
Boundary Conditions	180
Finite Element Formulation	181
Method of Numerical Solution	183
Numerical Results	183
Grid Dependency of the Flow Field	184
Diffuser Flow Field and Off-Design Performance	184
References	187
14 Extracting the Finite Element Domain from a Larger Flow System	189
Introduction	189
Analysis	191
Selection Options of the Computational Domain	191
Flow-Governing Equations	192
Boundary Conditions	193
Stage Inlet Station	193
Impeller Inlet and Exit Stations	193
Stage Exit Station	193

Solid Boundary Segments	193
Finite Element Formulation	194
Numerical Results	194
References	199
15 Finite Element Application to Unsteady Flow Problems	201
Introduction	201
Example	201
Flow-Governing Equations	204
Continuity Equation	204
Radial Momentum Equation	204
Tangential Momentum Equation	205
Axial Momentum Equation	205
Boundary Conditions	205
Finite Element Formulation	207
Time-Integration Algorithm	210
Numerical Procedure	211
Computational Results	211
Proposed Analysis Upgrades	219
Bidirectional Transfer of Boundary Conditions	219
Starting Point	220
Two-Way Stator/Rotor Exchange of Boundary Conditions	220
Continuity of the Variables' Normal Derivatives through Implicit Means	223
Methodology	223
Analysis	224
Problems	226
References	235
16 Finite Element–Based Perturbation Approach to Unsteady Flow Problems	237
Overview	237
Foundation of the Finite Element–Based Perturbation Approach	238
Definition of the Force-Related Rotordynamic Coefficients	240
Computational Development: Analysis of the Centered-Rotor Flow Field	242
Flow-Governing Equations	242
Continuity Equation	242
Axial Momentum Equation	242
Radial Momentum Equation	243
Tangential Momentum Equation	243
Boundary Conditions	243
Flow Inlet Station	243
Flow Exit Station	244
Solid Boundary Segments	244
Introduction of the Upwinding Technique	244

Finite Element Formulation	245
Method of Numerical Solution	248
Assessment of the Centered-Rotor Flow Field	249
Computational Development: Building the Zeroth-Order	
Flow Model	249
Strategy	251
Transition to an Alternate Frame of Reference	252
Adaptation of the Axisymmetric Flow Solution	253
Flow-Governing Equations in the Rotating Frame of Reference	255
Continuity Equation	255
x-Momentum Equation	255
y-Momentum Equation	255
z-Momentum Equation	255
Calculation of the Force-Related Rotordynamic Coefficients	256
Applications: Benchmark Test Case—Comparison with Cal Tech's	
Experimental Work	258
Background	258
Features of the Centered-Rotor Flow Field	259
Assessment of the Fluid-Induced Force Components	260
Applications: Perturbed Flow Structure due to Synchronous Whirl	262
Overview	262
Grid Dependency Investigation	264
Samples of the Computational Results	265
Comparison with Experimental Data	267
Applications: Rotordynamic Analysis of Labyrinth Seals	273
Literature Survey	273
Centered-Rotor Flow Field	275
Investigation of the Grid Dependency	278
Fluid-Induced Forces and Rotordynamic Coefficients	280
Applications: Rotordynamic Behavior of a Shrouded Pump Impeller	283
Centered-Impeller Subproblem: Contouring the Flow Domain	284
Centered-Impeller Subproblem: Boundary Conditions	284
Stage Inlet Station	284
Impeller Inlet and Exit Stations	285
Stage Exit Station	285
Solid Boundary Segments	285
Flow Structure	285
Simulation of the Impeller Subdomain Effects	291
Worthiness of Simulating the Impeller Subdomain	292
Results of the Perturbation Analysis	294
Assessment of the Single-Harmonic Perturbation Assumption	296
Applications: Investigation of Annular Seals under Conical Whirl	298
Rotordynamic Analysis of the Fluid/Rotor Interaction	
System	299
Computational Results	301
Applications: Interrelated Effects of the Cylindrical/Conical	
Rotor Whirl	303

Expanded Rotordynamic Analysis	304
Computational Results	306
Applications: Compressible-Flow Gas Seals Using a Simplified Adiabatic-Flow Approach	307
Computational Results	308
Comment	308
Proposed Upgrades of the Perturbation Analysis	309
Inclusion of the Shear-Stress Perturbations in Computing the Fluid-Induced Forces	309
Rigorous Adaptation to Compressible-Flow Problems	311
Relevant Remarks	311
Problems	313
References	317
<i>Appendix A. Natural Coordinates for Three-Dimensional Surface Elements</i>	321
<i>Appendix B. Classification and Finite Element Formulation of Viscous Flow Problems</i>	324
<i>Appendix C. Numerical Integration</i>	331
<i>Appendix D. Finite Element–Based Perturbation Analysis: Formulation of the Zeroth-Order Flow Field</i>	335
<i>Appendix E. Displaced-Rotor Operation: Perturbation Analysis</i>	344
<i>Appendix F. Rigorous Adaptation to Compressible-Flow Problems</i>	355
<i>Index</i>	369