

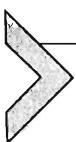
Continuum Theory for the Edge of an Open Lipid Bilayer

Aisa Biria, Mohsen Maleki, Eliot Fried

Department of Mechanical Engineering, McGill University, Montréal, Québec H3A 0C3, Canada

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A Variational Approach to Modeling Coupled Thermo-Mechanical Nonlinear Dissipative Behaviors

Laurent Stainier

Research Institute in Civil and Mechanical Engineering (GeM), Ecole Centrale Nantes, 1 rue de la Noë,
F-44321 Nantes, France

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