

# Contents

<i>Foreword to the Second Edition</i>	vii
<i>Foreword to the First Edition</i>	ix
1. A Brief Introduction to Turbulence	1
1.1 Common Features of Turbulent Flows . . . . .	1
1.1.1 Introductory concepts . . . . .	1
1.1.2 Randomness and coherent structure in turbulent flows . . . . .	3
1.2 Turbulent Scales and Complexity of a Turbulent Field . . . . .	5
1.2.1 Basic equations of turbulent flow . . . . .	5
1.2.2 Defining turbulent scales . . . . .	8
1.2.3 A glimpse at numerical simulations of turbulent flows . . . . .	14
1.3 Inter-scale Coupling in Turbulent Flows . . . . .	15
1.3.1 The energy cascade . . . . .	15
1.3.2 Inter-scale interactions . . . . .	17
2. Turbulence Simulation and Scale Separation	21
2.1 Numerical Simulation of Turbulent Flows . . . . .	21
2.2 Reducing the Cost of the Simulations . . . . .	23
2.2.1 Scale separation . . . . .	24
2.2.2 Navier–Stokes-based equations for the resolved quantities . . . . .	24
2.2.3 Navier–Stokes-based equations for the unresolved quantities . . . . .	26

2.3	The Averaging Approach: Reynolds-Averaged Numerical Simulation (RANS) . . . . .	26	3.5	Spectral Closures for Local Approaches . . . . .	71
2.3.1	Statistical average . . . . .	26	3.5.1	Local multiscale Reynolds stress models . . . . .	71
2.3.2	Reynolds-Averaged Navier–Stokes equations . . . . .	28	3.5.1.1	Closures for the linear transfer term . . . . .	72
2.3.3	Phase-Averaged Navier–Stokes equations . . . . .	29	3.5.1.2	Closures for the linear pressure term . . . . .	73
2.4	The Large-Eddy Simulation Approach (LES) . . . . .	31	3.5.1.3	Closures for the non-linear homogeneous transfer term . . . . .	74
2.4.1	Large and small scales separation . . . . .	31	3.5.1.4	Closures for the non-linear non-homogeneous transfer term . . . . .	76
2.4.2	Filtered Navier–Stokes equations . . . . .	33	3.5.2	Local multiscale eddy viscosity models . . . . .	77
2.5	Multilevel/Multiresolution Methods . . . . .	35	3.6	Achievements and Open Issues . . . . .	78
2.5.1	Hierarchical multilevel decomposition . . . . .	36	4.	Multiscale Subgrid Models: Self-adaptivity . . . . .	85
2.5.2	Practical example: the multiscale/multilevel LES decomposition . . . . .	38	4.1	Fundamentals of Subgrid Modelling . . . . .	85
2.5.3	Associated Navier–Stokes-based equations . . . . .	39	4.1.1	Functional and structural subgrid models . . . . .	85
2.5.4	Classification of existing multilevel methods . . . . .	41	4.1.2	The Gabor–Heisenberg curse . . . . .	86
2.5.4.1	Multilevel methods based on resolved-only wave numbers . . . . .	41	4.2	Germano-type Dynamic Subgrid Models . . . . .	91
2.5.4.2	Multilevel methods based on higher wave numbers . . . . .	42	4.2.1	Germano identity . . . . .	91
2.5.4.3	Adaptive multilevel methods . . . . .	43	4.2.1.1	Two-level multiplicative Germano identity . . . . .	91
2.6	Summary . . . . .	44	4.2.1.2	Multilevel Germano identity . . . . .	93
3.	Statistical Multiscale Modelling . . . . .	51	4.2.1.3	Generalized Germano identity . . . . .	94
3.1	General . . . . .	51	4.2.2	Derivation of dynamic subgrid models . . . . .	94
3.2	Exact Governing Equations for the Multiscale Problem . . . . .	54	4.2.3	Dynamic models and self-similarity . . . . .	97
3.2.1	Basic equations in physical and spectral space . . . . .	54	4.2.3.1	Turbulence self-similarity . . . . .	97
3.2.2	The multiscale splitting . . . . .	59	4.2.3.2	Scale separation operator self-similarity . . . . .	104
3.2.3	Governing equations for band-integrated approaches . . . . .	60	4.3	Self-Similarity Based Dynamic Subgrid Models . . . . .	106
3.3	Spectral Closures for Band-integrated Approaches . . . . .	62	4.3.1	Terracol–Sagaut procedure . . . . .	106
3.3.1	Local versus non-local transfers . . . . .	62	4.3.2	Shao procedure . . . . .	110
3.3.2	Expression for the spectral fluxes . . . . .	64	4.4	Variational Multiscale Methods and Related Subgrid Viscosity Models . . . . .	112
3.3.3	Dynamic spectral splitting . . . . .	67	4.4.1	Hughes VMS approach and extended formulations . . . . .	112
3.3.4	Turbulent diffusion terms . . . . .	68	4.4.2	Implementation of the scale separation operator . . . . .	117
3.3.5	Viscous dissipation term . . . . .	68	4.4.3	Bridging with hyperviscosity and filtered models . . . . .	120
3.3.6	Pressure term . . . . .	69			
3.4	A Few Multiscale Models for Band-integrated Approaches . . . . .	69			
3.4.1	Multiscale Reynolds stress models . . . . .	69			
3.4.2	Multiscale eddy viscosity models . . . . .	70			

5. Structural Multiscale Subgrid Models: Small Scales Estimations . . . . .	123	6.2.2 The dynamic multilevel (DML) method of dubois, jauberteau and temam . . . . .	193
5.1 Small-scale Reconstruction Methods: Deconvolution . . . . .	124	6.2.2.1 Spectral multilevel decomposition . . . . .	194
5.1.1 The velocity estimation model . . . . .	126	6.2.2.2 Associated Navier–Stokes-based equations . . . . .	196
5.1.2 The Approximate Deconvolution Model (ADM) . . . . .	132	6.2.2.3 Quasi-static approximation . . . . .	197
5.1.2.1 The original ADM approach of Stolz, Adams and Kleiser . . . . .	132	6.2.2.4 General description of the spectral multilevel method . . . . .	197
5.1.2.2 Example of application . . . . .	136	6.2.2.5 Dynamic estimation of the parameters $i_1$ , $i_2$ and $n_v$ . . . . .	199
5.1.2.3 Alternative formulation by explicit filtering . . . . .	140	6.2.3 Dynamic global multilevel LES . . . . .	201
5.1.2.4 Alternative regularization by standard subgrid models . . . . .	141	6.3 Adaptive Wavelet-based Methods: CVS, SCALES . . . . .	205
5.1.2.5 Relaxation-based approaches . . . . .	142	6.3.1 Wavelet decomposition: brief reminder . . . . .	206
5.1.2.6 Subgrid scales estimation by approximate deconvolution . . . . .	148	6.3.2 Coherency diagram of a turbulent field . . . . .	208
5.2 Small Scales Reconstruction: Multifractal Subgrid-scale Modelling . . . . .	150	6.3.2.1 Introduction to the coherency diagram . . . . .	208
5.2.1 General idea of the method . . . . .	150	6.3.2.2 Threshold value and error control . . . . .	210
5.2.2 Multifractal reconstruction of subgrid vorticity . . . . .	151	6.3.3 Adaptive wavelet-based direct numerical simulation . . . . .	213
5.2.2.1 Vorticity magnitude cascade . . . . .	152	6.3.4 Coherent vortex capturing method . . . . .	213
5.2.2.2 Vorticity orientation cascade . . . . .	154	6.3.5 Stochastic coherent adaptive large-eddy simulation . . . . .	214
5.2.2.3 Reconstruction of the subgrid velocity field . . . . .	155	6.4 DNS and LES with Optimal AMR . . . . .	219
5.3 Variational Multiscale Methods . . . . .	156	6.4.1 Error definition: surfacic versus volumic formulation . . . . .	219
5.4 Multigrid-based Decomposition . . . . .	157	6.4.2 <i>A posteriori</i> error estimation and optimization loop . . . . .	221
5.5 Global Multigrid Approaches: Cycling Methods . . . . .	161	6.4.3 Numerical results . . . . .	224
5.5.1 The multimesh method of Voke . . . . .	162	7. Global Hybrid RANS/LES Methods . . . . .	227
5.5.2 The multilevel LES method of Terracol <i>et al.</i> . . . . .	165	7.1 Bridging between Hybrid RANS/LES Methods and Multiscale Methods . . . . .	227
5.5.2.1 Cycling procedure . . . . .	165	7.1.1 Concept: the effective filter . . . . .	227
5.5.2.2 Multilevel subgrid closures . . . . .	167	7.1.2 Eddy viscosity effective filter . . . . .	229
5.5.2.3 Examples of application . . . . .	172	7.1.3 Global hybrid RANS/LES methods as multiscale methods . . . . .	231
5.6 Zonal Multigrid/Multidomain Methods . . . . .	174	7.2 Motivation and Classification of RANS/LES Methods . . . . .	232
6. Unsteady Turbulence Simulation on Self-adaptive Grids . . . . .	183		
6.1 Turbulence and Self-adaptivity: Expectations and Issues . . . . .	183		
6.2 Adaptive Multilevel DNS and LES . . . . .	188		
6.2.1 Dynamic local multilevel LES . . . . .	189		

7.3	Unsteady Statistical Modelling Approaches . . . . .	236
7.3.1	Unsteady RANS approach . . . . .	237
7.3.2	The Semi-Deterministic Method of Ha Minh . . . . .	240
7.3.3	The Scale Adaptive Simulation (SAS) . . . . .	246
7.3.4	The Turbulence-Resolving RANS approach of Travin <i>et al.</i> . . . . .	250
7.4	Global Hybrid Approaches . . . . .	252
7.4.1	The Approach of Speziale . . . . .	253
7.4.2	Limited Numerical Scales (LNS) . . . . .	257
7.4.2.1	General idea of LNS . . . . .	257
7.4.2.2	Example of application . . . . .	257
7.4.3	Blending methods . . . . .	258
7.4.3.1	General idea of blending methods . . . . .	258
7.4.3.2	Applications . . . . .	260
7.4.4	Other approaches: PITM and PANS . . . . .	263
7.4.4.1	Partially Integrated Transport Model (PITM) . . . . .	263
7.4.4.2	Partially Averaged Navier–Stokes (PANS) . . . . .	265
7.4.5	Detached Eddy Simulation . . . . .	266
7.4.5.1	General idea . . . . .	266
7.4.5.2	DES based on the SA model . . . . .	268
7.4.5.3	Possible extensions of standard SA-DES . . . . .	271
7.4.5.4	Examples . . . . .	273
7.4.5.5	DES based on the $k - \omega$ model . . . . .	273
7.4.5.6	Extra-Large Eddy Simulation (XLES) . . . . .	277
7.4.6	Grey-Area Modelled-Stress-Depletion (MSD) Grid and Induced Separation (GIS) . . . . .	279
7.4.7	Further interpretation of MSD: non-local error analysis . . . . .	283
7.4.8	Delayed Detached Eddy Simulation (DDES) . . . . .	286
7.4.8.1	Formulation . . . . .	286
7.4.8.2	Improved Delayed Detached Eddy Simulation (IDDES) . . . . .	289
7.4.9	Zonal Detached Eddy Simulation (ZDES) . . . . .	292
7.4.9.1	Formulation . . . . .	292
7.4.9.2	Implementation . . . . .	294
7.4.9.3	Interpretation and further discussion . . . . .	298

8.	Zonal RANS/LES Methods . . . . .	303
8.1	Theoretical Setting of RANS/LES Coupling . . . . .	305
8.1.1	Full-variables approach . . . . .	305
8.1.1.1	Enrichment procedure from RANS to LES . . . . .	307
8.1.1.2	Restriction procedure from LES to RANS . . . . .	309
8.1.2	Perturbation approach: NLDE . . . . .	310
8.2	Inlet Data Generation – Mapping Techniques . . . . .	314
8.2.1	Precursor calculation . . . . .	315
8.2.2	Recycling methods . . . . .	318
8.2.3	Main issues and possible improvements of recycling methods . . . . .	322
8.3	Synthetic Turbulence . . . . .	326
8.3.1	Spectral methods . . . . .	327
8.3.1.1	Inverse Fourier transform technique . . . . .	327
8.3.1.2	Random Fourier modes synthesization . . . . .	328
8.3.2	Digital filtering procedure by Klein–Sadiki–Janicka . . . . .	335
8.3.3	The Sandham–Yao–Lawal procedure . . . . .	337
8.3.4	Synthetic Eddy Method (SEM) . . . . .	340
8.3.4.1	The original procedure by Jarrin <i>et al.</i> . . . . .	340
8.3.4.2	Further improvements of the SEM method . . . . .	342
8.3.4.3	Adaptations to ZDES . . . . .	345
8.4	Forcing Methods . . . . .	348
8.4.1	The Spille–Kohoff–Kaltenbach controlled forcing method . . . . .	349
8.4.2	The dynamic forcing method by Laraufe–Deck–Sagaut . . . . .	353
8.4.3	The Dahlström and Davidson procedure . . . . .	356
9.	Feedback from Numerical Experiments . . . . .	361
9.1	Flow Physics Classification and Modelling Strategy Suitability . . . . .	361
9.2	Illustrative Examples . . . . .	367
9.2.1	Practical industrial applications . . . . .	367
9.2.2	Wall turbulence simulation . . . . .	369

9.3 Further Discussion . . . . .	371
9.3.1 Numerical discretization effects . . . . .	371
9.3.1.1 General statement . . . . .	371
9.3.1.2 Grid resolution requirements . . . . .	372
9.3.1.3 More detailed discussion: classical LES . . .	376
9.3.2 Zonal vs. non-zonal treatment of turbulence . . . .	391
<i>Bibliography</i>	397
<i>Index</i>	425