CONTENTS

1. WINGS 1

1.1. Function 1
1.2. Geometry 4
1.3. References 9
1.4. Problems 10

2. REVIEW OF BASIC FLUID DYNAMICS 11

2.1. Forces and Moments Due to Pressure 11
2.2. The Basic Conservation Laws of Fluid Mechanics 14
2.3. Vector Calculus 18
2.4. Differential Forms of the Conservation Laws 21
2.5. Rotational Velocity and Irrotational Flow 23
2.6. Two-Dimensional Incompressible Flow 25
2.6.1. Uniform Flow 27
2.6.2. Source Flow 27
2.6.3. Vortex Flow 28
2.7. Bibliography 29
2.8. Problems 29

3. INCOMPRESSIBLE IRROTATIONAL FLOW ABOUT SYMMETRIC AIRFOILS AT ZERO LIFT 32

3.1. Uniform Two-Dimensional Irrotational Incompressible Flow About an Isolated Body 33
3.2. Superposition of Fundamental Solutions 34

3.3. Dimensionless Variables 38
3.4. Rankine Ovals 39
3.5. Line Source Distributions 43
3.6. Flow Past Thin Symmetric Airfoils 46
3.7. Errors Near The Stagnation Points 52
3.8. Numerical Solution Based on Line Doublet Distributions 55
3.9. Relation of Numerical to Analytical Solutions 58
3.10. Complex–Variable Methods 60
3.10.1. Flow Past an Ellipse 63
3.10.2. Joukowsky Airfoils 66
3.11. Problems 70
3.12. Computer Programs 75

4. LIFTING AIRFOILS IN INCOMPRESSIBLE IRROTATIONAL FLOW 80

4.1. The Thin Airfoil: Thickness and Camber Problems 80
4.2. Forces and Moments on a Thin Airfoil 85
4.3. The Kutta Condition 91
4.4. Circulation Specification 93
4.5. The Cambered Thin Airfoil 94
4.6. Aerodynamics of The Thin Airfoil 97
4.7. The Lumped-Vortex Method 101
4.8. Panel Methods 103
4.8.1. Program PANEL 110
4.9. Complex-Variables Methods 112
4.10. References 117
4.11. Problems 117
4.12. Computer Program 118

5. WINGS OF FINITE SPAN 124

5.1. The Vortex System for a Thin Planar Wing of Finite Span 124
5.2. The Vortex–Lattice Method 130
5.3. Induced Drag 134
5.4. Lifting-Line Theory 135
5.5. The Elliptic Lift Distribution 138
5.6. The Optimal Wing 141
5.7. Nonelliptic Lift Distributions 144
5.8. References 148
5.9. Problems 149
5.10. Computer Program 151

6. THE NAVIER–STOKES EQUATIONS 154

6.1. The Stress at a Point 154
6.2. Newton's Second Law For Fluids 158
6.3. Symmetry of Stresses 160
6.4. Molecular View of Stress in a Fluid 161
6.5. The No-Slip Condition 166
6.6. Unidirectional Flows 168
6.7. The Viscosity Coefficient 170
6.8. Pascal's Law 171
6.9. Strain Versus Rotation 173
6.10. Isotropy 177
6.11. Vectors and Tensors 179
6.12. The Stress Tensor 182
6.13. The Rate-of-Strain Tensor 184
6.14. The Two Coefficients of Viscosity 185
6.15. The Navier–Stokes Equations 187
6.16. Problems 189

7. THE BOUNDARY LAYER 191

7.1. The Laminar Boundary Layer 191
7.2. Use of the Boundary-Layer Equations 197
7.2.1. Skin Friction 198
7.2.2. Displacement Thickness 199
7.2.3. Momentum Thickness 202
7.3. The Momentum Integral Equation 203
7.4. Velocity Profile Fitting: Laminar Boundary Layers 204
7.5. Thwaites's Method For Laminar Boundary Layers 206
7.6. Form Drag 209
7.7. Turbulent Flows 210
7.8. Velocity Profile Fitting: Turbulent Boundary Layers 215
7.9. Head's Method For Turbulent Boundary Layers 219
7.10. Transition From Laminar to Turbulent Flow 220
7.11. Boundary Layer Separation 222
7.12. Airfoil Performance Characteristics 225
7.13. The Development of Circulation About a Sharp-Tailed Airfoil 244
7.14. Computation of Boundary Layer Growth Along An Airfoil 247
7.15. References 252
7.16. Problems 253
7.17. Computer Program 255

8. PANEL METHODS 260

8.1. Mathematical Foundations: Green's Identity 260
8.2. Potential-Based Panel Methods 266

8.2.1. Constant-Potential Method 264
8.2.2. Linear-Potential Method 273
8.2.3. Equivalent Vortex Distributions 276
8.3. Vortex-Based Panel Methods 280
8.4. Source-Based Panel Methods 282
8.5. Comparisons of Source-, Doublet-, and Vortex-Based Methods 285
8.6. References 287
8.7. Problems 287

9. FINITE DIFFERENCE METHODS 289

9.1. Boundary-Value Problems in One Dimension 289
9.2. Convergence and Order of Accuracy 295
9.3. Incompressible Potential Flow Past a Thin Symmetric Airfoil 299
9.3.1. Direct Methods 303
9.3.2. Iterative Methods 304
9.4. Initial Problems: The Heat Equation 308
9.4.1. An Explicit Finite-Difference Method 310
9.4.2. Stability 312
9.4.3. Convergence 315
9.4.4. The Crank–Nicolson Method 317
9.4.5. Backward-Difference Schemes 320
9.5. References 322
9.6. Problems 322
9.7. Computer Programs 324

10. FINITE-DIFFERENCE SOLUTION OF THE BOUNDARY LAYER EQUATIONS 335

10.1. Statement of The Problem 335
10.2. Similar Solutions of The Laminar Incompressible Boundary Layer 337
10.2.1. Finite-Difference Methods for the Falkner–Skan Equation 341
10.2.2. Iterative Solution of Nonlinear Equations 342
10.2.3. A Finite-Difference Method Based on a Second-Order Differential Equation 344
10.2.4. A Finite-Difference Method Based on a System of First-Order Equations 348
10.3. Transformation of The Laminar Boundary-Layer Equations For Arbitrary Pressure Gradients 354
10.3.1. Program BDYLAY 359
10.4. Turbulent Boundary Layers 360
10.5. Separated Flows 366

10.6. References 368
10.7. Problems 369
10.8. Computer Programs 371

11. COMPRESSIBLE POTENTIAL FLOW PAST AIRFOILS 381

11.1. Shock Waves and Sound Waves 381
11.2. Equations of Compressible Steady Potential Flow 385
11.3. The Prandtl–Glauert Equation 388
11.4. Subsonic Flow Past Thin Airfoils 390
11.5. Supersonic Flow Past Thin Airfoils 393
11.6. Transonic Flow Past Thin Airfoils 399
11.6.1. Aerodynamics in the Transonic Range 402
11.6.2. Solution of the Transonic Small-Disturbance Equation: Subcritical Flow 404
11.6.3. Conservative versus Nonconservative Difference Schemes 406
11.6.4. Supercritical Flow and Upwind Differencing 408
11.6.5. The Relaxation Iteration 413
11.6.6. The Poisson Iteration 415
11.7. References 417
11.8. Problems 417
11.9. Computer Program 418

APPENDIX A. An Important Integral **424**

APPENDIX B. The Integral $I_n = \mathcal{P}\int_0^c \frac{t^n\,dt}{x-t}$ **427**

APPENDIX C. Potential Flow Past a Corner **429**

APPENDIX D. Uniqueness of Solutions of Laplace Equation **433**

APPENDIX E. Fourier-Series Expansions **438**

APPENDIX F. Downwash Due to a Horseshoe Vortex **442**

APPENDIX G. Geometrical Demonstration That Strain is a Tensor **446**

APPENDIX H. Optimization of the SOR Method for the Laplace Equation **449**

APPENDIX I. Structure of a Weak Shock Wave **456**

INDEX 459