Contents

	Preface to the first edition Preface to the second edition		page ix
			xiii
	Ackn	owledgements	xv
	PAR	T ONE TURBULENCE	1
1	Intro	oduction	3
	1.1	Turbulence	3
	1.2	Low-dimensional models	5
	1.3	The contents of this book	8
	1.4	Notation and mathematical jargon	11
2	Coh	erent structures	17
	2.1	Introduction	17
	2.2	Flows with coherent structures	21
	2.3	Detection of coherent structures	32
	2.4	The mixing layer	35
	2.5	The turbulent boundary layer	50
	2.6	A preview of things to come	65
3	Proj	per orthogonal decomposition	68
	3.1	Introduction	69
	3.2	On domains and averaging	73
	3.3	Properties of the POD	74
	3.4	Further results	86
	3.5	Stochastic estimation	91
	3.6	Coherent structures and homogeneity	93
	3.7	Some applications	96
	3.8	Appendix: some foundations	100
4	Gal	erkin projection	106
	4.1	Introduction	106
	4.2	Some simple PDEs revisited	110

vi Contents

	4.3 The Navier–Stokes equations	116
	4.4 Towards low-dimensional models	121
5	Balanced proper orthogonal decomposition	130
	5.1 Balanced truncation	131
	5.2 Balanced POD	133
	5.3 Output projection	136
	5.4 Connections with standard POD	137
	5.5 Extensions of balanced POD	139
	5.6 Some examples	143
	PART TWO DYNAMICAL SYSTEMS	153
6	Qualitative theory	155
	6.1 Linearization and invariant manifolds	156
	6.2 Periodic orbits and Poincaré maps	162
	6.3 Structural stability and genericity	165
	6.4 Bifurcations local and global	168
	6.5 Attractors simple and strange	179
7	Symmetry	190
	7.1 Equivariant vector fields	190
	7.2 Local bifurcation with symmetry	194
	7.3 Global behavior with symmetry	195
	7.4 An $O(2)$ -equivariant ODE	202
	7.5 Traveling modes	211
8	One-dimensional "turbulence"	214
	8.1 Projection onto Fourier modes	215
	8.2 Local bifurcations from $u = 0$	217
	8.3 The second bifurcation point	220
	8.4 Spatio-temporal chaos	226
9	Randomly perturbed systems	236
	9.1 An Ornstein–Uhlenbeck process	237
	9.2 Noisy heteroclinic cycles	240
	9.3 Power spectra of homoclinic attractors	247
	9.4 Symmetry breaking	249
	PART THREE THE BOUNDARY LAYER	253
10	Low-dimensional models	255
	10.1 Equations for coherent structures	256
	10.2 The eigenfunction expansion	259
	10.3 Symmetries	260
	10.4 Galerkin projection	262
	10.5 Geometrical structure of the model	260

Contents	vii
----------	-----

10.6	Choosing subspaces and domains	272
10.7	The energy budget	275
10.8	Nonlinear feedback	281
10.9	Interaction with unresolved modes	285
Beha	vior of the models	289
11.1	Backbones for the models	290
11.2	Heteroclinic cycles	293
11.3	Bursts and sweeps	297
11.4	The pressure term	299
11.5	More modes and instabilities	303
11.6	A tentative summary	307
11.7	Appendix: coefficients	312
PAR	T FOUR OTHER APPLICATIONS AND RELATED WORK	315
Some	e other fluid problems	317
12.1	The circular jet	317
12.2	The transitional boundary layer	321
12.3	A forced transitional mixing layer	326
12.4	Flows in complex geometries	328
12.5	"Full channel" wall layer models	331
12.6	Flows in internal combustion engines	335
12.7	A miscellany of results: 1995–2011	341
12.8	Discussion	342
. Revi	ew: prospects for rigor	345
13.1	The quality of models	345
13.2	A short-time tracking estimate	349
13.3	Stability, simulations, and statistics	352
13.4	Spatial localization	356
13.5	The utility of models	360
Refei	rences	364
	10.7 10.8 10.9 Beha 11.1 11.2 11.3 11.4 11.5 11.6 11.7 PAR Some 12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 . Revi 13.1 13.2 13.3 13.4 13.5	10.7 The energy budget 10.8 Nonlinear feedback 10.9 Interaction with unresolved modes Behavior of the models 11.1 Backbones for the models 11.2 Heteroclinic cycles 11.3 Bursts and sweeps 11.4 The pressure term 11.5 More modes and instabilities 11.6 A tentative summary 11.7 Appendix: coefficients PART FOUR OTHER APPLICATIONS AND RELATED WORK Some other fluid problems 12.1 The circular jet 12.2 The transitional boundary layer 12.3 A forced transitional mixing layer 12.4 Flows in complex geometries 12.5 "Full channel" wall layer models 12.6 Flows in internal combustion engines 12.7 A miscellany of results: 1995–2011 12.8 Discussion Review: prospects for rigor 13.1 The quality of models 13.2 A short-time tracking estimate 13.3 Stability, simulations, and statistics 13.4 Spatial localization