

Contents

Preface	v
General Notations	xiii
Chapter I. Stochastic canonical equation of multidimensional nonlinear dissipative Hamiltonian dynamical systems	1
1. Introduction, 1	
2. General equation of stochastic nonlinear dissipative Hamiltonian dynamical systems, 1	
3. General considerations on the kinetic energy, 3	
4. Modeling of the kinetic energy and mass matrix, 5	
5. Generalized potential, potential energy function and Lagrangian, 6	
6. Generalized momentum and Hamiltonian, 7	
7. Energy function, 8	
8. Total energy of the dynamical system, 9	
9. Model of the Hamiltonian, 9	
10. Generalized momentum and generalized velocity, 11	
11. Modeling of the nonconservative generalized force, 12	
12. Quasi-linear term in the generalized velocity, 13	
13. Quasi-linear damping term, 13	
14. Quasi-linear gyroscopic term, 15	
15. External stochastic excitation, 16	
16. Stochastic parametric excitation term, 17	
Chapter II. Fundamental examples of nonlinear dynamical systems and associated second-order equation	19
1. Introduction, 19	
2. Second-order equation associated with the dissipative Hamiltonian dynamical system, 19	

3. Examples for multidimensional nonlinear oscillators under external random excitations, 21	
4. Examples for multidimensional nonlinear oscillators under parametric and external random excitations, 27	
5. Remark concerning modeling of hysteresis loops, 34	
6. Complements on the potential energy function, 36	
Chapter III. Brief review of probability and random variables	43
1. Introduction, 43	
2. Principles of probability and random variables, 43	
3. Random variable with values in \mathbb{R}^n , 45	
4. Second-order random variable with values in \mathbb{R}^n and $L^2(\mathcal{A}, \mathbb{R}^n)$ -space, 48	
5. Classical examples of probability laws, 50	
6. Transformations of random variables with values in \mathbb{R}^n , 51	
7. Convergence of a sequence of random variables with values in \mathbb{R}^n , 54	
Chapter IV. Probabilistic tools I. Classical stochastic processes	57
1. Introduction, 57	
2. Basic concepts and definition of classical stochastic processes, 57	
3. Continuity of stochastic processes, 62	
4. Markov processes, 64	
5. Second-order stochastic processes with values in \mathbb{R}^n , 69	
6. Spectral analysis of second-order mean-square stationary stochastic processes mean-square continuous with values in \mathbb{R}^n , 73	
7. Mean-square derivative of second-order stochastic processes with values in \mathbb{R}^n , 77	
8. Mean-square integral of second-order stochastic processes with values in \mathbb{R}^n , 81	
Chapter V. Probabilistic tools II. Mean-square theory of linear integral transformations and of linear differential equations	83
1. Introduction, 83	
2. Integral linear transformation with function-kernel of second-order stochastic processes, 84	
3. Linear filtering and spectral analysis of second-order mean-square continuous and mean-square stationary stochastic processes, 88	
4. Normalized Gaussian white noise, 93	
5. Linear filtering of normalized Gaussian white noise, 95	
6. Physically realizable processes, 97	

7. Mean-square theory of ordinary linear stochastic differential equations, 98	
Chapter VI. Probabilistic tools III. Diffusion processes and Fokker-Planck equation	109
1. Introduction, 109	
2. Definition of a diffusion process, 110	
3. Fokker-Planck equation for a diffusion process, 112	
4. Stationary Markov processes, 118	
5. Stationary diffusion processes, 120	
6. Calculation of second-order quantities of a stationary diffusion processes, 123	
Chapter VII. Probabilistic tools IV. Stochastic integrals and stochastic differential equations	131
1. Introduction, 131	
2. Need for introducing stochastic integrals and their different types, 132	
3. Itô stochastic integral, 140	
4. Itô stochastic differentials, 144	
5. Itô stochastic differential equation, 149	
6. Stratonovich stochastic integral and Stratonovich stochastic differential equation, 158	
Chapter VIII. Stochastic modeling with stochastic differential equations	165
1. Introduction, 165	
2. Markov realization of physically realizable processes, 166	
3. Modeling with an SDE : Case of a Gaussian white noise, 173	
4. Modeling with an SDE : Case of a Gaussian process that has a finite dimensional Markov realization, 174	
5. Modeling with an SDE : Case of a physically realizable stationary Gaussian process, 175	
6. Modeling with an SDE : Case of a second-order stationary Gaussian wide-band physical noise, 176	
Chapter IX. FKP equation for the dissipative Hamiltonian dynamical systems	179
1. Introduction, 179	
2. Stochastic model of external and parametric excitations, 179	

3. Stochastic modeling with a Stratonovich stochastic differential equation, 180	
4. Transformation of the SSDE into an Itô stochastic differential equation, 182	
5. On the type of stochastic solution sought, 183	
6. Fokker-Planck equation for the stochastic dynamical system, 185	
Chapter X. Stationary response of dissipative dynamical systems, existence and uniqueness, explicit solution of an invariant measure	189
1. Introduction, 189	
2. Comments on the methodology followed in this chapter, 190	
3. Properties of the diffusion matrix, 194	
4. Preliminary mathematical results for the uniqueness theorems, 196	
5. Uniqueness of the invariant measure as a solution of the steady state FKP equation, 203	
6. Illustration of the use of the uniqueness theorem, 207	
7. Existence and uniqueness of an invariant measure as a solution of the steady state FKP equation, 210	
8. Existence and uniqueness of a stationary solution, 215	
Chapter XI. Complements for the normalization condition, characteristic function and moments of the invariant measure	217
1. Introduction, 217	
2. Dynamical systems considered, 217	
3. Calculation of the constant of normalization, 220	
4. Calculation of the characteristic function, 222	
5. Calculation of second-order moments, 226	
6. Total energy stochastic process, 229	
Chapter XII. Application I. Multidimensional linear oscillators subject to external and parametric random excitations	231
1. Introduction, 231	
2. Results concerning special functions useful for the developments, 231	
3. Multidimensional linear oscillator subject to external and parametric random excitations, 233	
4. Formulas for multidimensional linear oscillators subject to external random excitations, 239	
Chapter XIII. Application II. Multidimensional nonlinear oscillators with inertial nonlinearity subject to external random excitations	243
1. Introduction, 243	

2. Multidimensional nonlinear oscillator with inertial nonlinearity, 244	
3. Characteristic function of r.v. $(\mathbf{Q}(t), \dot{\mathbf{Q}}(t))$, 248	
4. Second-order moments of r.v. $(\mathbf{Q}(t), \dot{\mathbf{Q}}(t))$, 253	
5. Monte Carlo numerical simulation, 256	
6. Application, 257	
Chapter XIV. Application III. Multidimensional nonlinear oscillators subject to external and parametric random excitations	265
1. Introduction, 265	
2. Examples for multidimensional nonlinear oscillators subject to external random excitations, 265	
3. Examples for multidimensional nonlinear oscillators subject to parametric and external random excitations, 272	
Chapter XV. Symplectic change of variables in the multidimensional unsteady FKP equation	277
1. Introduction, 277	
2. Unsteady FKP equation, 278	
3. Symplectic matrix, 279	
4. Symplectic transformations, 282	
5. Poisson bracket with a symplectic change of variables, 289	
6. Symplectic change of variables in the FKP equation, 290	
7. Application: A nonstationary solution of the unsteady FKP equation using infinitesimal symplectic transformation, 292	
References	301
Index	315