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Mixing and Chaos in Open Flows

A. MOURA^a, U. FEUDEL^b, and E. GOUILLART^c

^a*Institute of Complex Systems and Mathematical Biology, King's College, University of Aberdeen, Aberdeen AB24 3UE, UK; e-mail: a.moura@abdn.ac.uk*

^b*Institut für Chemie und Biologie des Meeres, Carl von Ossietzky Universität, PF 2503, D-26111 Oldenburg, Germany*

^c*Surface du Verre et Interfaces, UMR 125 CNRS/Saint-Gobain, 93303 Aubervilliers, France*

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Abstract

We review the theoretical and experimental knowl-
ing chaotic advection, from a point of view of dyna-
mic systems theory. The unstable manifold of the
chaotic saddle and its stable and unstable manifolds constitute the skeleton around which

The Role of Discontinuities in Mixing

R. STURMAN

Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, United Kingdom

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Abstract

Chaotic advection in fluids is traditionally associated with stretching and folding. Although this is an effective and widely used technique to produce efficient mixing, it is not

On Dissipation in Stirred Mixtures

E. VILLERMAUX^{a,b}

^a*Aix Marseille Université, IRPHE, 13384 Marseille Cedex 13, France*

^b*Institut Universitaire de France, Paris, France*

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Beyond Passive: Chaotic Transport in Stirred Fluids

G. METCALFE^a, M.F.M. SPEETJENS^b, D.R. LESTER^c,
and H.J.H. CLERCX^d

^a*Commonwealth Scientific & Industrial Research Organization (CSIRO), Division of Materials Science & Engineering, PO Box 56, Highett, Victoria 3190, Australia*

^b*Department of Mechanical Engineering, Energy Technology, Eindhoven University of Technology, PO Box 513, MB Eindhoven 5600, The Netherlands*

^c*Commonwealth Scientific & Industrial Research Organization (CSIRO), Division of Mathematics, Informatics & Statistics, PO Box 56, Highett, Victoria 3190, Australia*

^d*Department of Applied Physics, Fluid Dynamics Laboratory, Eindhoven University of Technology, PO Box 513, MB Eindhoven 5600, The Netherlands*

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Chaotic Advection for Thermal Mixing

Y. LE GUER and K. EL OMARI

*Laboratoire des Sciences de l'Ingénieur Appliquées
à la Mécanique et au Génie Electrique (SIAME) – Fédération IPRA
Université de Pau et des Pays de l'Adour (UPPA)
Bat. d'Alembert, rue Jules Ferry,
BP. 7511, 64075 Pau, France*

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