Contents

Preface *ix*
Foreword *xi*
Future Contributions *xv*

1. Beam Equations **1**

1.1. Introduction 2
1.2. Some Facts from the Tensor Analysis 3
1.3. Elements of Differential Geometry of the Coordinate Surfaces 10
1.4. Tensor Form of the Beam Equations 15
1.5. Beam Equations in the Orthogonal Coordinate System 21
1.6. Geometrized Beam Equations 25
1.7. Geometrized Equations for Two-Dimensional Beams 33
1.8. Theory of the Single-Component Flows 37

2. Exact Solutions to the Beam Equations **45**

2.1. Introduction 46
2.2. Some Facts from the Theory of Continuous Groups 47
2.3. Calculation of the Main Group for the Equations Describing a One-Dimensional Nonstationary Beam 53
2.4. Group Properties of the Beam Equations 61
2.5. Invariant Solutions 66
2.6. Optimal Systems of the Subgroups for a Three-Dimensional Stationary Beam 68
2.7. Results of Constructing the Invariant Solutions 78
2.8. Solutions Invariant with Respect to the Transformations with Arbitrary Functions of Time 84
2.9. Invariant Solutions of the Geometrized Beam Equations 111
2.10. The Exact Solutions, Whose Relation to the Group Properties Is Yet Unknown 115

3. Antiparaxial Expansions **123**

3.1. Introduction 124
3.2. Solutions of the Equations for Potential Electrostatic Flows in ρ-Mode 126

3.3. Solutions of the Vortex Relativistic Beam Equations 135
3.4. Constructing the Solution in the Form of an Asymptotic Series with Respect to a Small Geometric Parameter 144
3.5. Extending the Working Area of Expansions 154
3.6. Coordinate Expansions in the Three-Dimensional Beam Formation Problem 171
3.7. Solution of the Beam Formation Problem in the Vicinity of the Arbitrary Axisymmetric Emitter 174
3.8. Differential Geometry of Trajectories in Three-Dimensional Flows 191

4. Solution of the Beam Formation Problem in Three Dimensions 199

4.1. Introduction 201
4.2. Brief Review of the Results of 2D Theory 202
4.3. Riemann Method 204
4.4. Formation of the Cylindrical Beam with Arbitrary Cross Section and Rectilinear Trajectories 209
4.5. Near-Emitter Region for a Cylindrical Beam 215
4.6. Forming Electrodes for Arbitrary Cylinder in *T*- and *U*-Modes 221
4.7. The Domains with Axial Symmetry and the Arbitrary Cone 226
4.8. Asymptotic Series for the Riemann Function for an Arbitrary Cone and Arbitrary Surface of Revolution 231
4.9. The Near-Emitter Solution 236
4.10. Solution with the Riemann Function in the Power Series Form 258
4.11. Some Computational Aspects of Solving the External Problem 267
4.12. On the Role of Conformal Mappings in the External Problem of the Beam Formation Theory 269
4.13. Calculation Results for Cylindrical Beams with Elliptical and Rectangular Cross Sections 280

5. Asymptotic Theory of 3D Flows 287

5.1. Introduction 290
5.2. Paraxial Beam Equations 290
5.3. 3D Beams with Homogeneously Deformed Cross Section 297
5.4. Expressions for Magnetic Field and Potential 299
5.5. Special Cases of Homogeneous Deformation and Analytical Solutions of Paraxial Equations 304
5.6. Regularization of Paraxial Equations 308
5.7. 3D Beams with Round Cross Section 320
5.8. Regularization of Quasi-Axisymmetric Flows 329
5.9. Formation of the Beams with a Rectangular Cross Section in Paraxial Approximation 342
5.10. The Near-Surface Beam Equations 350

5.11. Regularization of the Near-Surface Expansions 360
5.12. Tubular Relativistic Beams 367
5.13. Exact Solutions of the Near-Surface Beam Equation 373
5.14. Evolution of Paraxial Beam in a Given Electromagnetic Field 378

6. Geometrized Theory 389

6.1. Introduction 392
6.2. Antiparaxial Expansions in the 3D Case 394
6.3. First Statement of the Cauchy Problem for 2D Flows 405
6.4. Second Statement of the Cauchy Problem for 2D Flows 410
6.5. The Problem of Integrating the Geometrized Beam Equations 431
6.6. Approximate Solution of the Geometrized Beam Equations Using the Narrow Bands Method 437
6.7. Synthesis of the Solid Non-Paraxial Flows 448
6.8. Test Problems 461
6.9. Refined Model of the Near-Axis Region 464
6.10. Refined Model of the Near-Emitter Region 477
6.11. Geometrized Theory of the Non-Twisted Relativistic Beams 481
6.12. Theory of Third Approximation for an Internal Beam Surface as a Basic Stream Tube 500
6.13. Geometrized Theory of 3D Narrow Relativistic Beams 509
6.14. The Second-Order Approximation Theory for 3D Electrostatic Flows 515
6.15. Theory of the Narrow Vortex Relativistic Beams with Elliptical Cross Section 535

7. Examples of Applications 583

7.1. Introduction 585
7.2. Testing the Geometrized Models 586
7.3. A Model of the Cathode Vicinity 629
7.4. The Expansion Gap Problem and a Non-Pierce Slope Angle of the Zero-Potential Forming Electrode 635
7.5. Theory of the Multibeam Electron Optical Systems 647
7.6. The Magnetron-Injection Gun Model 666
7.7. Optimization of the Electron Flow Parameters on the Collector 691

References 699

Contents of Volumes 151-165 715

Index 721