

is treated in more detail. In Chap. 11 the different experimental techniques for the investigation of atoms and molecules are presented, in order to give the reader a feeling for the inventive ideas and the necessary experimental skill for their realization. The last chapter presents a short overview on recent developments in atomic and molecular physics, which shall demonstrate that physics will be never a complete and finalized field. There is still much to explore and new ideas and scientific enthusiasm is needed, to push the border of our knowledge further ahead. Some examples in this chapter also illustrate possible important applications of new ideas such as the quantum computer or new techniques of frequency metrology used in the world wide global positioning system GPS.

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This book is an extended version of volume 3 of a German textbook consisting of 4 volumes. The authors hopes, that it will find a comparable good acceptance as the German version. He will be grateful for any reply of readers, giving corrections of possible errors or hints to improvements. Any of such replies will be answered as soon as possible. A textbook lives from the active collaboration of its readers and the author looks foreward to a lively correspondence with his readers. He hopes that this book can contribute to a better understanding of this fascinating field of atoms, molecules and photons.

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