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Useful Vector Identities and other formulas	inside back cover

Chapter 1

Static Electric and Magnetic Fields in Vacuum

1.1 Static Charges

Static electricity, produced by rubbing different materials against one another, was known to the early Greeks who gave it its name (derived from *ἤλεκτρον*, pronounced *ēlektron*, meaning amber). Experiments by Du Fay in the early 18th century established that there are two kinds of electricity, one produced by rubbing substances such as hard rubber and amber, called resinous, and another produced by rubbing glassy substances such as quartz, dubbed vitreous. Objects with like charge were found to repel one another, while objects with unlike charge were found to attract. Benjamin Franklin attempted to explain electricity in terms of an excess or deficiency of the vitreous electric fluid, leading to the designations *positive* and *negative*.

A report by Benjamin Franklin that a cork ball inside an electrically charged metal cup is not attracted to the inside surface of the cup led Joseph Priestly to infer that, like gravity, electrical forces obey an inverse square law. This hypothesis was almost immediately confirmed (to limited accuracy) by John Robison, but the results were not published for almost 50 years. Cavendish, in an elegant experiment, showed that if a power law holds,¹ the exponent of r in the force law could not differ from minus two by more than 1 part in 50, but he failed to publish his results. Charles Augustin de Coulomb, who, in the late 18th century, measured both the attractive and repulsive force between charges with a delicate torsion balance, is credited with the discovery of the force law bearing his name – he found that the force is proportional to the product of the charges, acts along the line joining the charges, and decreases inversely as the square of the distance between them. Charges of opposite sign attract one another, whereas charges of the same sign repel. It has been verified experimentally that the exponent of r varies from minus two by no more than 1 part in 10^{16} over distances of order one meter.

¹A modern interpretation suggests that a test of the exponent is not appropriate because a power law is not the anticipated form. In line with considerations by Proca and Yukawa, the potential should take the form $e^{-\beta r}/r$ ($\beta = m_\gamma c/\hbar$) where m_γ is the rest mass (if any) of the photon. Astronomical measurements of Jupiter's magnetic field place an upper limit of 4×10^{-51} kg on the mass of the photon