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Preface

In this update of the 1975 book, we have added extensive material on industrial and warm dense matter plasmas, energetic particle measurements and instrumentation, collective Thomson scattering, and techniques. We have also expanded the discussion on relativistic theory and added an appendix on computation.

The form of the spectrum of electromagnetic radiation scattered by a charged particle depends on the mass, charge, position, and velocity of the particle. Similarly, the spectrum of radiation scattered by a plasma, an assembly of free charges, depends on the properties of the plasma. By suitable choice of a source of radiation, we can in principle probe any plasmas and measure quantities such as electron and ion densities and temperatures, ionic charge, magnetic fields, wave amplitudes, and plasma instabilities. It is of particular importance that these measurements can usually be made without significant perturbation of the plasma. In order that we may isolate the effect of the scattering, we require an essentially monochromatic source. Because the scattering cross section is small, the source must be powerful. The growing importance, over the years, of scattering as a diagnostic technique has been associated to a great extent with the development of lasers and microwave sources that satisfy these requirements.

In this monograph, we have attempted to cover both theory and experiment in a consistent and detailed fashion. We have made an effort toward providing quantitative results in which units are clearly defined, and there are no 4π 's and c 's lurking in the background. We review the underlying mathematical techniques.

The book is biased on the experimental side in the direction of laboratory high-temperature plasmas because this is our field of research. To illustrate the application of the technique, we describe a number of significant laboratory experiments that have been made over the last few decades. Scattering is a basic tool of the plasma physicist, and to help those who are not spectroscopists, we have included a section on the most commonly used dispersion instrumentation.

This text is written as an introduction text and reference book for graduate students and researchers. We hope that the readers will find answers to their questions at the level to which they are prepared to go.