Contents

	Perr	nissions		vi	
	Pref	Preface			
		Difficu	lties in the traditional approach	xiii	
			s of modern tools	XV	
		Structu	re of the book	xvi	
		Prerequ	nisites and remarks	xviii	
1	Basi	c physic	al setting	1	
	1.1		ginal N-body system	1	
		1.1.1	Physical context	1	
		1.1.2	The plasma model	2	
	1.2	Wave-1	particle resonance: a paradigm of classical mechanics	5	
2	Fron	n <i>N-</i> boo	ly dynamics to wave–particle interaction	10	
	2.1	Intuitiv	e derivation of the self-consistent Hamiltonian	11	
	2.2	Langm	uir waves without resonant particles	13	
		2.2.1	Decomposition of the field and particle motion—relevant		
			small parameters	13	
		2.2.2	Collective dynamics	15	
		2.2.3	Bohm-Gross modes	17	
	2.3	Couple	d motion of quasi-resonant particles with Bohm-Gross		
		modes		20	
	2.4	Lagran	gian formulation*	24	
	2.5	Referei	nce states of the plasma*	28	
		2.5.1	Non-existence of zero-field states	28	
		2.5.2	Thermal distribution of the electric field	29	
	2.6	Physica	al scalings and error estimates*	31	
	2.7		orm of the Hamiltonian	37	
	2.8	Historia	cal background and notes	38	

viii	Contents
7	COLLEGIA

3	Dyn	amics of the small-amplitude wave–particle system	40	
	3.1	Reference state with a vanishing electric field	41	5.4
	3.2	Small perturbation to the reference state	42	
	3.3	Ballistic solutions	46	
	3.4	Wavelike solutions	47	
	3.5	Initial value problem	49	
	3.6	Dispersion relation for wavelike modes	51	5.5
	3.7	Physical interpretation: cold beams	52	3.3
		3.7.1 Case of a single beam	52	
		3.7.2 Case of two beams	55	
	3.8	Many cold beams or a warm beam	57	
		3.8.1 Landau unstable mode	57	5.6
		3.8.2 Eigenmodes and initial value problem	59	Dif
		3.8.3 Van Kampen modes*	61	6.1
		3.8.4 Relation with ballistic eigenmodes	68	
		3.8.5 Transition from cold to warm beams	69	6.2
	3.9	Synchronization of particles with a wave	69	6.3
		3.9.1 Synchronization of particles with a wavelike mode	69	
		3.9.2 Synchronization of particles during Landau damping*	71	
		3.9.3 Fate of particles in the presence of many incoherent modes	72	
	3.10	Historical background	72	6.4
4	Stati	stical description of the small-amplitude wave–particle dynamics	74	6.5
	4.1	Approach using perturbation expansion	75	
		4.1.1 Second-order perturbation analysis	75	
		4.1.2 Evolution of waves	77	
		4.1.3 Evolution of particles	81	6.6
		4.1.4 Fokker–Planck equation for the particles	82	6.7
	4.2	Approach using Floquet equation*	85	6.8
	4.3	Link with traditional descriptions	90	
		4.3.1 Landau effect from a Vlasovian point of view	90	
		4.3.2 Spontaneous emission	92	6.9
_	YT	•		6.1
5		niltonian chaos	94	
	5.1	Geometrical tools for Hamiltonian chaos	96	6.1
		5.1.1 Poincaré surface of section	96	6.1
		5.1.2 Conservation of areas, symplectic dynamics and flux*	99	Sel
	<i>-</i> 0	$\boldsymbol{\mathcal{C}}$	101	7.1
	5.2	Motion of one particle in the presence of two waves	103	7.2
		5.2.1 Small resonance overlap and cantori	104	7.3
		5.2.2 Moderate resonance overlap and stochastic layers	106	
	~ ~	5.2.3 Physical summary	109	7.4
	5.3	Construction of orbits	111	7.5
		5.3.1 Origin of higher-order resonances	111	7.6
		5.3.2 Poincaré and KAM theorems*	114	7.7

		Contents	ix
	5.3.3	Higher-order resonances from action-angle variables	s 115
5.4	Renor	malization for KAM tori	117
	5.4.1	Simple approach to renormalization	117
	5.4.2	More explicit derivation*	119
	5.4.3	Study of the renormalization mapping	123
	5.4.4	Thresholds and exponents	125
5.5	Order	in chaos	129
	5.5.1	Wiggling arms of the X-point	129
	5.5.2	Large resonance overlap: numerical results	133
	5.5.3	Large resonance overlap: analytical results*	136
5.6	Histori	ical background and further comments	139
Diffu	ısion: t	he case of the non-self-consistent dynamics	141
6.1	Model	Hamiltonian	142
6.2	Diffusi	ion as a numerical fact	142
6.3	Conce	pt of resonance box	148
	6.3.1	Heuristic analytical approach	149
	6.3.2	Numerical check of the concept	153
	6.3.3	Rigorous approach*	154
6.4	Scaling	g properties of finite-time dynamics	156
6.5	Origin	of the force decorrelation	158
	6.5.1	Locality and resonance boxes for random phases	158
	6.5.2	Correlated phases	160
	6.5.3	Random initial positions	161
6.6	Origin	of the chaotic diffusion	161
6.7	Locali	ty and diffusion for more general Hamiltonians	163
6.8	Initial	quasilinear diffusion	166
	6.8.1	1	167
	6.8.2	Initial quasilinear diffusion up to chaos	169
6.9		ion coefficient in the chaotic regime	174
	_	oefficient	177
		r–Planck equation	181
6.12	Histori	ical background	182
		ent dynamics in the diffusive regime	183
7.1	-	inear diffusion coefficient	185
7.2	-	e derivation of the quasilinear equations	187
7.3		ion of waves*	189
7.4	_	on particles*	195
7.5		volution of particles and waves	197
7.6		tion of the weak-warm-beam instability	198
7.7	Histori	ical background and further comments	203

X	Contents
Λ	Comens

8	Tim	e evolution of the single-wave–particle system	205
	8.1	Self-consistent nonlinear regime with a single wave	205
		8.1.1 Instability saturation	206
		8.1.2 Nonlinear regime of wave damping	207
	8.2	Unstable beam–wave system	208
		8.2.1 Cold-beam—wave instability	208
		8.2.2 Landau instability	210
		8.2.3 Comparison with the Vlasov-wave description	215
	8.3	Linear and nonlinear wave damping	217
		8.3.1 Evolution of the wave intensity	217
		8.3.2 Finite- <i>N</i> effects competing with Landau damping	218
	8.4	Historical background and further comments	220
9	Gibbsian equilibrium of the single-wave-particle system*		
	9.1	Gibbs microcanonical and canonical ensembles	225
	9.2	Partition function	227
		9.2.1 Thermodynamic discussion	228
		9.2.2 Weight function	229
	9.3	High-temperature phase	230
		9.3.1 Thermodynamics and wave intensity	230
		9.3.2 Particle distribution and joint probabilities	231
	9.4	Low-temperature phase	233
		9.4.1 Thermodynamics and wave intensity	233
		9.4.2 Particle distribution and joint probabilities	234
	9.5	Phase diagrams	235
	9.6	Bernstein-Greene-Kruskal state	236
	9.7	Comparison with numerical simulations	238
	9.8	Further comments	240
10	Conc	clusion	241
		Summary	241
	10.2	Open issues	243
A	Cont	tinuous and discrete Fourier transforms	245
	A. 1	Perturbed lattices and discrete Fourier transforms	245
	A.2	Representation of trigonometric functions	246
	A.3	Laplace and Fourier transforms	247
В	Spec	cial functions	249
	B .1	Euler gamma function	249
	B.2	Bessel functions	250
	B.3	Dirac distribution	251
C	Phas	se space structure and orbits	254

		Contents	xi
D	D.1	plectic structure and numerical integration Symplectic dynamics* Numerical integration schemes Symplectic integration of Hamiltonian dynamics	256 256 257 258
E	E.1 E.2	Dability and stochastic processes General random variables and vectors Gaussian distributions Fokker–Planck equation	261 261 262 263
F	F.1 F.2 F.3 F.4 F.5	mates for chapter 6 n -time correlation function of $C(\tau)$ and $S(\tau)$ Estimate of the mean number of visited boxes Initial Brownian motion Dependence of an orbit on one phase Estimate of non-quasilinear terms	268 268 270 271 274 278
G	G.1 G.2	Vlasov-wave partial differential system G.1.1 Formulation G.1.2 Derivation from <i>N</i> -body dynamics: kinetic limit G.1.3 Fourier-transformed Vlasov-wave system Zero-field solutions and their linear stability G.2.1 Wavelike modes G.2.2 Ballistic modes G.2.3 Complete solution	282 283 283 285 288 288 289 290 290
	G.3 G.4	Bernstein–Greene–Kruskal modes Vlasov–Poisson system	291 292
	References		295
	Inde	X	305