

Contents

Preface	ix
I Introduction	1
1 Introduction	1
2 First order ODE	3
3 Classification and properties	9
4 Higher order ODE	13
5 Difference equations	15
6 Discretisations	18
Exercises	21
II Existence, Uniqueness and Dependence on Parameters	25
1 Lipschitz continuity and uniqueness	25
2 Local existence	30
3 Continuation of local solutions	34
4 Dependence on initial value and vector field	37
5 Regular perturbations; linearisation	43
Exercises	47
III Numerical Analysis of One-Step Methods	51
1 Runge-Kutta methods	51
2 Local discretisation errors; consistency	54
3 Convergence of one-step methods	57
4 Asymptotic expansion of the global error	60
5 Practical error estimation	62
6 Step size control	65
Exercises	75
IV Linear Systems	79
1 Introduction	79
2 The homogeneous case	82

vi	CONTENTS	CONTENTS	vii
3 The inhomogenous case	85	5 A -stable, $A(\alpha)$ -stable methods	216
4 Constant coefficients	85	6 BDF methods and their implementation	221
5 Periodic coefficients	88	Exercises	228
6 Planar systems	92		
7 Classification of planar, autonomous systems	98		
8 Difference equations	101	IX Differential-Algebraic Equations	231
Exercises	103	1 Introduction	231
V Stability	107	2 Linear DAE	234
1 Introduction	107	3 General DAE	237
2 Stability definitions	109	4 Semi-explicit DAE	240
3 Stability of linear systems	112	5 BDF methods for DAE	243
4 Nonlinear systems; linearisation	116	6 Regularisation and stabilisation	248
5 Nonlinear systems; Lyapunov functions	121	Exercises	252
6 Global analysis of the phase	125		
7 Periodic ODE	129	X Boundary Value Problems	257
8 Stability of Δ -equations	132	1 Introduction	257
Exercises	134	2 Existence of solutions	259
VI Chaotic Systems	137	3 Well-conditioning and dichotomy	261
1 Introduction	137	4 Single shooting	264
2 Local divergence	141	5 Single shooting for nonlinear problems	267
3 Lyapunov exponents	145	6 Multiple shooting	271
4 Strange and chaotic attractors	151	Exercises	275
5 Fractal dimension	157		
6 Reconstruction	161	XI Concepts from Classical Mechanics	279
7 Prediction	164	1 Introduction	279
Exercises	167	2 Lagrangian formalism	282
VII Numerical Analysis of Multistep Methods	171	3 Hamiltonian formalism	288
1 Linear multistep methods	171	4 Phase plane analysis	292
2 Consistency; local errors	174	5 Continuous media	296
3 Root stability	176	Exercises	299
4 Convergence	180		
5 Asymptotic expansion of the global error	184	XII Mathematical Modelling	303
6 Predictor-corrector techniques	185	1 Introduction	303
7 Variable step; variable order algorithms	189	2 Dimensional analysis	307
Exercises	196	3 Rear-wheel steering	315
VIII Singular Perturbations and Stiff Differential Equations	199	4 Inverse resonance	321
1 Singular perturbations	199	5 Solitary water waves	330
2 Matched asymptotic expansions	206	6 Infectious diseases I	337
3 Stiff differential equations	212	7 Infectious diseases II	344
4 The increment function	215	8 Nerve conduction	350
		9 Torsion in a crank shaft	359
		10 The dripping faucet	368
		Appendices	379
		References	399
		Index	403