

# Contents

Preface	ix
Chapter 1. Introduction	1
1.1 Example: Bending an Elastic Rod I	2
1.2 Principle of Linearization	5
1.3 Global Theory	6
1.4 Layout	7
<b>PART 1. LINEAR AND NONLINEAR FUNCTIONAL ANALYSIS</b>	<b>9</b>
Chapter 2. Linear Functional Analysis	11
2.1 Preliminaries and Notation	11
2.2 Subspaces	13
2.3 Dual Spaces	14
2.4 Linear Operators	15
2.5 Neumann Series	16
2.6 Projections and Subspaces	17
2.7 Compact and Fredholm Operators	18
2.8 Notes on Sources	20
Chapter 3. Calculus in Banach Spaces	21
3.1 Fréchet Differentiation	21
3.2 Higher Derivatives	27
3.3 Taylor's Theorem	31
3.4 Gradient Operators	32
3.5 Inverse and Implicit Function Theorems	35
3.6 Perturbation of a Simple Eigenvalue	38
3.7 Notes on Sources	40
Chapter 4. Multilinear and Analytic Operators	41
4.1 Bounded Multilinear Operators	41
4.2 Faà de Bruno Formula	44
4.3 Analytic Operators	45
4.4 Analytic Functions of Two Variables	52

4.5 Analytic Inverse and Implicit Function Theorems	53
4.6 Notes on Sources	57
<b>PART 2. ANALYTIC VARIETIES</b>	<b>59</b>
<b>Chapter 5. Analytic Functions on <math>\mathbb{F}^n</math></b>	<b>61</b>
5.1 Preliminaries	61
5.2 Weierstrass Division Theorem	64
5.3 Weierstrass Preparation Theorem	65
5.4 Riemann Extension Theorem	66
5.5 Notes on Sources	69
<b>Chapter 6. Polynomials</b>	<b>70</b>
6.1 Constant Coefficients	70
6.2 Variable Coefficients	74
6.3 Notes on Sources	77
<b>Chapter 7. Analytic Varieties</b>	<b>78</b>
7.1 $\mathbb{F}$ -Analytic Varieties	78
7.2 Weierstrass Analytic Varieties	81
7.3 Analytic Germs and Subspaces	86
7.4 Germs of $\mathbb{C}$ -analytic Varieties	88
7.5 One-dimensional Branches	95
7.6 Notes on Sources	99
<b>PART 3. BIFURCATION THEORY</b>	<b>101</b>
<b>Chapter 8. Local Bifurcation Theory</b>	<b>103</b>
8.1 A Necessary Condition	103
8.2 Lyapunov-Schmidt Reduction	104
8.3 Crandall-Rabinowitz Transversality	105
8.4 Bifurcation from a Simple Eigenvalue	109
8.5 Bending an Elastic Rod II	111
8.6 Bifurcation of Periodic Solutions	112
8.7 Notes on Sources	113
<b>Chapter 9. Global Bifurcation Theory</b>	<b>114</b>
9.1 Global One-Dimensional Branches	114
9.2 Global Analytic Bifurcation in Cones	120
9.3 Bending an Elastic Rod III	121
9.4 Notes on Sources	124

<b>PART 4. STOKES WAVES</b>	<b>125</b>
<b>Chapter 10. Steady Periodic Water Waves</b>	<b>127</b>
10.1 Euler Equations	127
10.2 One-dimensional Formulation	131
10.3 Main Equation	137
10.4 <i>A Priori</i> Bounds and Nekrasov's Equation	140
10.5 Weak Solutions Are Classical	146
10.6 Notes on Sources	151
<b>Chapter 11. Global Existence of Stokes Waves</b>	<b>152</b>
11.1 Local Bifurcation Theory	152
11.2 Global Bifurcation from $\lambda = 1$	154
11.3 Gradients, Morse Index and Bifurcation	157
11.4 Notes on Sources	159
<b>Bibliography</b>	<b>161</b>
<b>Index</b>	<b>167</b>