

Contents

Preface	xiii
1. ON REGULARIZATION OF LINEAR EQUATIONS ON THE BASIS OF PERTURBATION THEORY	1
1. Generalised Jordan chains, sets and root numbers of linear operators	1
2. Regularization of linear equations with Fredholm operators	6
3. Principal theorem on regularization of linear equations by the perturbation method	14
4. Regularization of linear equations on the basis of perturbation theory in Hilbert spaces	21
5. Computation of eigenvalues and eigenvectors of linear operators by pseudo-perturbation method	28
6. Notes	42
2. INVESTIGATION OF BIFURCATION POINTS OF A NONLINEAR EQUATIONS	43
1. Lyapunov–Schmidt BEq in the problem of a bifurcation point	45
2. General existence theorems for the bifurcation points	50
3. Construction of asymptotics in a neighborhood of a bifurcation point	63
4. Asymptotic bifurcation points	87
5. On perturbation of the branch points of nonlinear equations	92
6. Notes and generalization	94
3. REGULARIZATION OF COMPUTATION OF SOLUTIONS IN A NEIGHBORHOOD OF THE BRANCH POINT	99
1. Construction of the regularizing equation in the problem at a branch point	103
2. Definition and properties of simple solutions	121
3. Regularization of calculations of simple solutions of nonlinear equations	134

4.	Regularization of method for continuation along parameter in a neighborhood of a branch point	142	3.2	Potential BEq	256
5.	Notes and remarks	149	4.	Direct methods of BEq group invariance usage for its general form construction by admitted group symmetry	259
4.	ITERATIONS, INTERLACED EQUATIONS AND LYAPUNOV CONVEX MAJORANTS IN NONLINEAR ANALYSIS	151	4.1	Applications of Lie–Ovsyannikov theorem about invariant manifolds for the construction of BEq general form by allowing group symmetry	270
1.	Iterations and uniformization of branching solutions in nonlinear analysis	151	5.	Non-linearly perturbed Helmholtz equations	279
1.1	BEq and the selection of the initial approximation	152	5.1	Domain symmetry and bifurcational solutions asymptotics	279
1.2	On the role of supporting lines and Newton diagrams in the construction of the initial approximation	157	5.2	Periodic solutions	286
1.3	A one-step iteration method	159	6.	Capillary–gravity waves in fluid layers	295
1.4	An N -step iteration method	163	6.1	Capillary–gravity waves in a floating fluid spatial layer	296
1.5	On regularization in the sense of Tikhonov, modifications and possible generalizations of an N -step method	168	6.2	Capillary–gravity waves at the interface of two fluids flow	299
1.6	Remarks	172	6.3	Capillary–gravity waves on a cylindrical surface	299
2.	Interlaced and potential BEq	172	6.4	Ferrofluid layer in a magnetic field	301
2.1	The property of (S, K) -interlacing of an equation and its inheritance by the BEq	174	7.	Fluid phase state crystallization problem in statistical crystal theory	302
2.2	(T, M) -interlaced and (T^2, M) -interlaced BEq	178	7.1	The statement of the problem	302
2.3	α -parametric interlaced BEq	180	7.2	Subspaces $N(B_s)$. Their expansions on irreducible subspaces relative to O_h	306
2.4	Interlaced BEq of potential type	183	7.3	The BEq construction	311
2.5	Surface bundle of a domain of free parameters	189	7.4	Asymptotics of small solutions families for $n_s = 1, 3$	315
2.6	Parametrization of solutions and the method of successive approximations	195	7.5	Solutions invariant relative to normal divisors O_h	317
3.	On the role of Lyapunov convex majorants in the nonlocal existence theorems of implicit functions	198	8.	Andronov–Hopf bifurcation under group symmetry conditions	320
3.1	Majorants independent of parameters	200	8.1	BEq derivation in non-stationary bifurcation	321
3.2	Majorants depending on a parameter	209	8.2	Symmetry inheritance theorem	323
3.3	Investigation of the existence domain of the solution of equation $F(u, \varepsilon) = 0$.	213	8.3	BEq construction by group analysis methods	325
5.	METHODS OF REPRESENTATION THEORY AND GROUP ANALYSIS IN BIFURCATION THEORY	217	8.4	On the asymptotics of small solutions	328
1.	Nonlinear equations invariant under transformation groups	218	9.	Stability of the bifurcating solutions	330
1.1	Lyapounov–Schmidt BEq and some methods of their reduction	218	6.	SINGULAR DIFFERENTIAL EQUATIONS IN BANACH SPACES	337
1.2	Some applications	223	1.	Fundamental operator functions	338
2.	Hereditary symmetry of branching equations and resolving systems	226	1.1	Generalized functions in Banach spaces	339
2.1	Invariance properties of BEq	226	1.2	Fundamental operator functions of singular differential operators	343
2.2	Resolving systems for differential equations with Fredholm operator at the derivative and their symmetry	232	1.3	Fundamental operator functions of singular integral and integro-differential operators	357
2.3	On the Grobman–Hartman theorem for equations with degenerate operator at the derivative	246	2.	The initial value problem for a differential equation having a Noetherian operator at the derivative. Periodic solutions and the property of convergence	362
3.	Construction and investigation of the branching equation by group analysis methods	252	2.1	Auxiliary information on Jordan sets of Noetherian operators	362
3.1	BEq of solutions invariant relative to subgroups of the original equation group symmetry	252	2.2	The initial value problem for a linear differential equation	364
			2.3	The initial value problem for a nonlinear differential equation	368
			2.4	Periodic solutions	373

2.5	Integral pseudo-solutions	376
3.	Non-stationary differential equations with singularities	383
3.1	The initial value problem for a non-stationary linear equation and systems of 1st kind integral Volterra equations	383
3.2	The initial value problem for a non-stationary linear differential equation and a system of integral Volterra equations with a singularity	393
3.3	Branching differential equations of the initial value problem with singularity	397
3.4	The initial value problem for a nonlinear differential equation and equations with a singular point	404
4.	Partial differential equations with the Fredholm operator in the main part	413
5.	The theory of semigroups and groups of operators with kernels	419
5.1	Relative resolvents. Relatively adjoint elements	419
5.2	Relatively spectrally bounded operators and analytical groups of operators with kernels	421
5.3	Relatively sectorial operators and analytical semigroups of operators with kernels	428
7.	STEADY-STATE SOLUTIONS OF THE VLASOV-MAXWELL SYSTEM	431
1.	Introduction	431
2.	Stationary solutions of a VM system	434
2.1	The reduction of VM system to the resolving elliptic system (2.28), (2.29)	435
2.2	The reduction of the resolving system to the unique resolving equation	439
2.3	Existence of solutions of the boundary value problem (2.40)–(2.42)	443
2.4	Applications of the reduction theorems	449
2.5	Normalized solutions for a one-component distribution function	462
3.	Non-stationary problem	468
4.	Bifurcation points and nontrivial solutions of the stationary VM system	474
4.1	Introduction	474
4.2	Statement of the boundary value problem and the problem at a bifurcation point	475
4.3	Resolution of the bifurcation equation	485
4.4	The existence theorem for bifurcation points and the construction of asymptotic solutions	487
	Appendices	497
A–	Positive solutions of the nonlinear singular boundary value problem of magnetic insulation	497
	References	513