

Contents

Part I. Basic Experimental Facts and Theoretical Tools	
<hr/>	
1. Introduction	3
1.1 Goal	3
1.2 Brain: Structure and Functioning. A Brief Reminder	4
1.3 Network Models	5
1.4 How We Will Proceed	6
2. The Neuron – Building Block of the Brain	9
2.1 Structure and Basic Functions	9
2.2 Information Transmission in an Axon	10
2.3 Neural Code	12
2.4 Synapses – The Local Contacts	13
2.5 Naka–Rushton Relation	14
2.6 Learning and Memory	16
2.7 The Role of Dendrites	16
3. Neuronal Cooperativity	17
3.1 Structural Organization	17
3.2 Global Functional Studies.	
Location of Activity Centers	23
3.3 Interlude: A Minicourse on Correlations	25
3.4 Mesoscopic Neuronal Cooperativity	31
4. Spikes, Phases, Noise:	
How to Describe Them Mathematically?	
We Learn a Few Tricks and Some Important Concepts	37
4.1 The δ -Function and Its Properties	37
4.2 Perturbed Step Functions	43
4.3 Some More Technical Considerations*	46
4.4 Kicks	48
4.5 Many Kicks	51
4.6 Random Kicks or a Look at Soccer Games	52

4.7	Noise Is Inevitable. Brownian Motion and the Langevin Equation	54
4.8	Noise in Active Systems	56
4.8.1	Introductory Remarks	56
4.8.2	Two-State Systems	57
4.8.3	Many Two-State Systems: Many Ion Channels	58
4.9	The Concept of Phase.....	60
4.9.1	Some Elementary Considerations	60
4.9.2	Regular Spike Trains.....	63
4.9.3	How to Determine Phases From Experimental Data? Hilbert Transform	64
4.10	Phase Noise	68
4.11	Origin of Phase Noise*	71

Part II. Spiking in Neural Nets

5.	The Lighthouse Model. Two Coupled Neurons.....	77
5.1	Formulation of the Model.....	77
5.2	Basic Equations for the Phases of Two Coupled Neurons ...	80
5.3	Two Neurons: Solution of the Phase-Locked State	82
5.4	Frequency Pulling and Mutual Activation of Two Neurons ...	86
5.5	Stability Equations	89
5.6	Phase Relaxation and the Impact of Noise	94
5.7	Delay Between Two Neurons	98
5.8	An Alternative Interpretation of the Lighthouse Model	100
6.	The Lighthouse Model. Many Coupled Neurons	103
6.1	The Basic Equations	103
6.2	A Special Case. Equal Sensory Inputs. No Delay.....	105
6.3	A Further Special Case. Different Sensory Inputs, but No Delay and No Fluctuations	107
6.4	Associative Memory and Pattern Filter.....	109
6.5	Weak Associative Memory. General Case*	113
6.6	The Phase-Locked State of N Neurons. Two Delay Times ...	116
6.7	Stability of the Phase-Locked State. Two Delay Times*	118
6.8	Many Different Delay Times*	123
6.9	Phase Waves in a Two-Dimensional Neural Sheet	124
6.10	Stability Limits of Phase-Locked State	125
6.11	Phase Noise*	126
6.12	Strong Coupling Limit. The Nonsteady Phase-Locked State of Many Neurons	130
6.13	Fully Nonlinear Treatment of the Phase-Locked State*	134

7.	Integrate and Fire Models (IFM)	141
7.1	The General Equations of IFM	141
7.2	Peskin's Model	143
7.3	A Model with Long Relaxation Times of Synaptic and Dendritic Responses	145
8.	Many Neurons, General Case, Connection with Integrate and Fire Model	151
8.1	Introductory Remarks.....	151
8.2	Basic Equations Including Delay and Noise	151
8.3	Response of Dendritic Currents	153
8.4	The Phase-Locked State	155
8.5	Stability of the Phase-Locked State: Eigenvalue Equations ...	156
8.6	Example of the Solution of an Eigenvalue Equation of the Form of (8.59).....	159
8.7	Stability of Phase-Locked State I: The Eigenvalues of the Lighthouse Model with $\gamma' \neq 0$	161
8.8	Stability of Phase-Locked State II: The Eigenvalues of the Integrate and Fire Model	162
8.9	Generalization to Several Delay Times	165
8.10	Time-Dependent Sensory Inputs.....	166
8.11	Impact of Noise and Delay	167
8.12	Partial Phase Locking.....	167
8.13	Derivation of Pulse-Averaged Equations	168

Appendix 1 to Chap. 8: Evaluation of (8.35)	173
----------------------------------------------------------	------------

Appendix 2 to Chap. 8: Fractal Derivatives	177
---------------------------------------------------------	------------

**Part III. Phase Locking, Coordination
and Spatio-Temporal Patterns**

9.	Phase Locking via Sinusoidal Couplings.....	183
9.1	Coupling Between Two Neurons	183
9.2	A Chain of Coupled-Phase Oscillators.....	186
9.3	Coupled Finger Movements	188
9.4	Quadruped Motion	191
9.5	Populations of Neural Phase Oscillators	193
9.5.1	Synchronization Patterns	193
9.5.2	Pulse Stimulation	193
9.5.3	Periodic Stimulation	194

10. Pulse-Averaged Equations 195

10.1 Survey 195

10.2 The Wilson–Cowan Equations. 196

10.3 A Simple Example. 197

10.4 Cortical Dynamics Described by Wilson–Cowan Equations . . . 202

10.5 Visual Hallucinations 204

10.6 Jirsa–Haken–Nunez Equations 205

10.7 An Application to Movement Control 209

10.7.1 The Kelso Experiment 209

10.7.2 The Sensory-Motor Feedback Loop 211

10.7.3 The Field Equation and Projection onto Modes 212

10.7.4 Some Conclusions 213

Part IV. Conclusion

11. The Single Neuron 217

11.1 Hodgkin–Huxley Equations 217

11.2 FitzHugh–Nagumo Equations 218

11.3 Some Generalizations of the Hodgkin–Huxley Equations 222

11.4 Dynamical Classes of Neurons. 223

11.5 Some Conclusions on Network Models. 224

12. Conclusion and Outlook 225

References 229

Index 241