

# Contents

<b>Preface</b>	<b>v</b>
<b>Introduction</b>	<b>1</b>
<b>I Basic spectral methods</b>	<b>7</b>
<b>1 Fundamentals of spectral methods</b>	<b>9</b>
1.1 Generalities on the method of weighted residuals . . . . .	9
1.2 Approximation of a given function . . . . .	11
1.2.1 Galerkin-type method . . . . .	11
1.2.2 Collocation method . . . . .	12
1.3 Approximation of the solution of a differential equation . .	12
1.3.1 The traditional Galerkin method . . . . .	13
1.3.2 The tau method . . . . .	14
1.3.3 The collocation method . . . . .	14
<b>2 Fourier Method</b>	<b>17</b>
2.1 Truncated Fourier series . . . . .	17
2.1.1 Calculation of Fourier coefficients . . . . .	18
2.1.2 Some results on convergence . . . . .	19
2.2 Discrete Fourier series . . . . .	20
2.3 Relation between Galerkin and collocation coefficients . .	21
2.4 Odd and even collocation . . . . .	22

2.5	Differentiation in the physical space . . . . .	23
2.6	Differential equation with constant coefficients . . . . .	25
2.6.1	Galerkin method . . . . .	25
2.6.2	Collocation method . . . . .	26
2.7	Differential equation with variable coefficients . . . . .	28
2.7.1	Galerkin method . . . . .	28
2.7.2	Collocation method . . . . .	28
2.8	Nonlinear differential equation . . . . .	31
2.9	Aliasing removal . . . . .	33
<b>3</b>	<b>Chebyshev method</b>	<b>39</b>
3.1	Generalities on Chebyshev polynomials . . . . .	40
3.2	Truncated Chebyshev series . . . . .	43
3.2.1	Calculation of Chebyshev coefficients . . . . .	43
3.2.2	Differentiation . . . . .	44
3.3	Discrete Chebyshev series and collocation . . . . .	46
3.3.1	Calculation of Chebyshev coefficients . . . . .	46
3.3.2	Relation between collocation and Galerkin coefficients . . . . .	48
3.3.3	Lagrange interpolation polynomial . . . . .	49
3.3.4	Differentiation in the physical space . . . . .	50
3.3.5	Round-off errors . . . . .	51
3.3.6	Relationship with finite-difference and similar approximations . . . . .	54
3.4	Differential equation with constant coefficients . . . . .	55
3.4.1	Tau method . . . . .	56
3.4.2	Collocation method . . . . .	59
3.4.3	Error equation . . . . .	62
3.5	Differential equation with nonconstant coefficients . . . . .	65
3.5.1	Linear equation with variable coefficients . . . . .	65
3.5.2	Nonlinear equation . . . . .	65
3.5.3	Aliasing . . . . .	70
3.6	Some results of convergence . . . . .	70
3.7	Multidimensional elliptic equation . . . . .	77
3.7.1	One-dimensional equation . . . . .	78
3.7.2	Two-dimensional equation . . . . .	88
3.7.3	Three-dimensional equation . . . . .	93
3.8	Iterative solution methods . . . . .	98
<b>4</b>	<b>Time-dependent equations</b>	<b>101</b>
4.1	Introduction . . . . .	101
4.2	The advection-diffusion equation . . . . .	104
4.2.1	The exact initial-boundary value problem . . . . .	104
4.2.2	Fourier approximation . . . . .	105
4.2.3	Chebyshev approximation . . . . .	106

4.3	One-step method: the weighted two-level scheme . . . . .	110
4.3.1	Fourier approximation . . . . .	111
4.3.2	Chebyshev approximation . . . . .	111
4.4	Two-step methods . . . . .	114
4.4.1	Fourier approximation . . . . .	115
4.4.2	Chebyshev approximation . . . . .	119
4.4.3	Numerical illustration . . . . .	125
4.5	High-order time-discretization methods . . . . .	130
4.5.1	Multistep methods . . . . .	130
4.5.2	One-step methods . . . . .	142
4.5.3	Conclusion . . . . .	153
<b>II</b>	<b>Navier-Stokes equations</b>	<b>155</b>
<b>5</b>	<b>Navier-Stokes equations for incompressible fluids</b>	<b>157</b>
5.1	Velocity-pressure equations . . . . .	157
5.2	Vorticity-streamfunction equations . . . . .	159
5.2.1	Plane flow . . . . .	159
5.2.2	Axisymmetric flow . . . . .	162
5.3	Boussinesq approximation . . . . .	163
5.4	Semi-infinite domain . . . . .	164
<b>6</b>	<b>Vorticity-Streamfunction Equations</b>	<b>167</b>
6.1	Introduction . . . . .	167
6.2	Fourier-Fourier method . . . . .	168
6.3	Fourier-Chebyshev method . . . . .	169
6.3.1	Flow in a plane channel . . . . .	169
6.3.2	Time-dependent Stokes equations . . . . .	170
6.3.3	Navier-Stokes equations . . . . .	184
6.3.4	Case of nonzero flow rate . . . . .	186
6.4	Chebyshev-Chebyshev method . . . . .	188
6.4.1	Time-discretization . . . . .	188
6.4.2	The influence matrix method . . . . .	188
6.4.3	Other influence matrix methods . . . . .	195
6.5	Examples of application . . . . .	195
6.5.1	Rayleigh-Bénard convection . . . . .	196
6.5.2	Axisymmetric flow in a rotating annulus . . . . .	202
<b>7</b>	<b>Velocity-Pressure Equations</b>	<b>207</b>
7.1	Introduction . . . . .	207
7.2	Fourier-Fourier-Fourier method . . . . .	208
7.2.1	Time-discretization schemes . . . . .	209
7.2.2	The nonlinear term . . . . .	210
7.3	Fourier-Chebyshev method . . . . .	212

7.3.1	Flow in a plane channel . . . . .	213
7.3.2	Time-dependent Stokes equations . . . . .	213
7.3.3	Navier-Stokes equations . . . . .	251
7.4	Chebyshev-Chebyshev method . . . . .	254
7.4.1	The influence matrix method . . . . .	255
7.4.2	The projection method . . . . .	266
7.4.3	Shear-stress-free boundary . . . . .	277
7.4.4	Assessment of methods . . . . .	278
7.4.5	The form of the nonlinear term . . . . .	286
7.5	Example of application: three-dimensional flow in a rotating annulus . . . . .	289
<b>III</b>	<b>Special topics</b>	<b>295</b>
<b>8</b>	<b>Stiff and singular problems</b>	<b>297</b>
8.1	Introduction . . . . .	298
8.2	Stiff problems : coordinate transformation approach . . . . .	300
8.2.1	Requirements . . . . .	300
8.2.2	Some typical mappings . . . . .	301
8.2.3	Self-adaptive coordinate transformation . . . . .	307
8.3	Stiff problem: domain decomposition approach . . . . .	314
8.3.1	Fixed subdomains and nonadapted coordinate systems . . . . .	314
8.3.2	Fixed subdomains and adapted coordinate transformations . . . . .	316
8.3.3	Adapted subdomains and nonadapted coordinate systems . . . . .	316
8.3.4	Adapted subdomains and adapted coordinate transformations . . . . .	317
8.4	Singular problems . . . . .	321
8.4.1	Singular solution of the Laplace equation . . . . .	323
8.4.2	Navier-Stokes equations . . . . .	328
8.5	Filtering technique . . . . .	335
<b>9</b>	<b>Domain Decomposition Method</b>	<b>339</b>
9.1	Introduction . . . . .	339
9.2	Differential equation . . . . .	341
9.2.1	Patching method: two-domain solution . . . . .	341
9.2.2	Patching method : multidomain solution . . . . .	346
9.2.3	Spectral-element method . . . . .	348
9.2.4	Numerical illustration . . . . .	352
9.3	Two-dimensional Helmholtz equation . . . . .	354
9.3.1	Patching method . . . . .	355
9.3.2	Influence matrix method . . . . .	357

9.3.3	Schur complement problem . . . . .	360
9.3.4	Identity between influence matrix and Schur complement . . . . .	361
9.4	Time-dependent equations . . . . .	362
9.4.1	Stability of the Chebyshev approximation . . . . .	362
9.4.2	Stability of time-discretization schemes . . . . .	363
9.5	Navier-Stokes equations in vorticity-streamfunction variables . . . . .	365
9.5.1	Fourier-Chebyshev method . . . . .	365
9.5.2	Chebyshev-Chebyshev method . . . . .	368
9.5.3	Examples of application . . . . .	371
9.6	Navier-Stokes equation in velocity-pressure variables . . . . .	375
9.6.1	Stokes problem : Fourier-Chebyshev method . . . . .	376
9.6.2	Stokes problem: Chebyshev-Chebyshev method . . . . .	378
9.6.3	Example of application . . . . .	385
<b>Appendix A Formulas on Chebyshev polynomials</b>		<b>389</b>
A.1	Definition and general properties . . . . .	389
A.2	Differentiation . . . . .	390
A.3	Collocation points . . . . .	391
A.4	Truncated series expansion . . . . .	392
A.5	Lagrange interpolation polynomial . . . . .	393
A.6	Derivatives at Gauss-Lobatto points . . . . .	393
A.7	Integration . . . . .	394
A.8	Numerical integration based on Gauss-Lobatto points . . . . .	394
<b>Appendix B Solution of a quasi-tridiagonal system</b>		<b>397</b>
<b>Appendix C Theorems on the zeros of a polynomial</b>		<b>399</b>
<b>References</b>		<b>401</b>
<b>Index</b>		<b>425</b>