Contents

Preface Acknowledgments	<i>page</i> ix				
Chapter I Introduction and Overview of Turbulence	1				
Introduction	1				
1. Viscous Fluids. The Navier–Stokes Equations					
2. Turbulence: Where the Interests of Engineers and Mathematicians					
Overlap					
3. Elements of the Theories of Turbulence of Kolmogorov and					
Kraichnan	ç				
4. Function Spaces, Functional Inequalities, and Dimensional Analysis	14				
Chapter II Elements of the Mathematical Theory of the					
Navier-Stokes Equations	25				
Introduction	25				
1. Energy and Enstrophy	27				
2. Boundary Value Problems	29				
3. Helmholtz-Leray Decomposition of Vector Fields	36				
4. Weak Formulation of the Navier-Stokes Equations	39				
5. Function Spaces	41				
6. The Stokes Operator	49				
7. Existence and Uniqueness of Solutions: The Main Results	55				
8. Analyticity in Time	62				
9. Gevrey Class Regularity and the Decay of the Fourier Coefficients	67				
10. Function Spaces for the Whole-Space Case	75				
11. The No-Slip Case with Moving Boundaries	77				
12. Dissipation Rate of Flows	80				
13. Nondimensional Estimates and the Grashof Number	87				
Appendix A. Mathematical Complements	90				
Appendix B. Proofs of Technical Results in Chapter II	102				

vii

viii Contents

Chapter III Finite Dimensionality of Flows	115				
Introduction					
1. Determining Modes					
2. Determining Nodes					
3. Attractors and Their Fractal Dimension					
4. Approximate Inertial Manifolds	150				
Appendix A. Proofs of Technical Results in Chapter III	156				
Chapter IV Stationary Statistical Solutions of the Navier-Stokes					
Equations, Time Averages, and Attractors	169				
Introduction	169				
1. Mathematical Framework, Definition of Stationary Statistical					
Solutions, and Banach Generalized Limits	172				
2. Invariant Measures and Stationary Statistical Solutions in					
Dimension 2	183				
3. Stationary Statistical Solutions in Dimension 3					
4. Attractors and Stationary Statistical Solutions	194				
5. Average Transfer of Energy and the Cascades in Turbulent Flows					
Appendix A. New Concepts and Results Used in Chapter IV					
Appendix B. Proofs of Technical Results in Chapter IV					
Appendix C. A Mathematical Complement: The Accretivity Property					
in Dimension 3	244				
Chapter V Time-Dependent Statistical Solutions of the					
Navier-Stokes Equations and Fully Developed					
Turbulence	255				
Introduction	255				
1. Time-Dependent Statistical Solutions on Bounded Domains	262				
2. Homogeneous Statistical Solutions					
3. Reynolds Equation for the Average Flow	280				
4. Self-Similar Homogeneous Statistical Solutions					
5. Relation with and Application to the Conventional Theory of					
Turbulence	295				
6. Some Concluding Remarks	310				
Appendix A. Proofs of Technical Results in Chapter V					
References	331				
Index	343				