## Contents

	by Fritz John	•
1.	Equations of Hyperbolic and Parabolic Types	•
2.	The Wave Operator	
	2.1. The one-dimensional wave equation	
	2.2. The initial value problem for the wave equation in three-sp	ace
	2.3. Analysis of the solution	•
	2.4. The method of descent	
	2.5. The inhomogeneous wave equation	
	2.6. The Cauchy problem for general initial surfaces	•
	2.7. Energy integrals and a priori estimates	
	2.8. The general linear equation with the wave operator	as
	principal part $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$	
	2.9. Mixed problems	•
	Discontinuities         .	
	<ul> <li>3.2. Relations between partial derivatives on a surface</li> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	•
4	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi-
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi-
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi- ions
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi- ions
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi- ions
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi- ions
4.	<ul> <li>3.3. Free surfaces. Characteristic matrix</li></ul>	effi- ions

xiii

## CONTENTS

5.	A Parabolic Equation: The Equation of Heat Cond	ucti	on			94
	5.1. Parabolic equations in general					94
	<ul><li>5.1. Parabolic equations in general</li><li>5.2. The heat equation. Maximum principle</li></ul>					96
	5.3. Solution of the initial value problem					98
	5.4. Smoothness of solutions					101
	5.5. The boundary initial value problem for a rectan	gle				104
-				-		
6.	Approximation of Solutions of Partial Differential	Equ	atio	ns l	эγ	
	the Method of Finite Differences	•	•	•		108
	6.1. Solution of parabolic equations		•	•	•	109
	6.2. Stability of difference schemes for other types	of	equa	atio	ns	115
	Bibliography	•	•	•	•	123
Part	II. Elliptic Equations,					
1 41 0	by LIPMAN BERS and MARTIN SCHECHTER					131
	by Diffman Ders and MARTIN SCHECHTER	•	•	•	•	151
1.	Elliptic Equations and Their Solutions					133
	1.1. Introduction					131
	1.2. Linear elliptic equations					134
	1.3. Smoothness of solutions					135
	1.4. Unique continuation					139
	1.5. Boundary conditions					
	Appendix I. Elliptic versus Strongly Elliptic					
	Appendix II. "Weak Equals Strong"					144
•						
2.	The Maximum Principle		•	•	•	150
	2.1. Second-order equations			•	•	150
	2.2. Statement and proof of the maximum principle	•	• *	•	•	150
	2.3. Applications to the Dirichlet problem	•	•	•	•	
	2.4. Applications to the generalized Neumann proble	em	•	•	•	154
	2.5. Solution of the Dirichlet problem by finite differ				•	155
	2.6. Solution of the difference equation by iterations			•	·	158
	2.7. A maximum principle for gradients		•	•	•	160
	2.8. Carleman's unique continuation theorem	•	•	•	•	162
3	Hilbert Space Approach. Periodic Solutions					164
	3.1. Periodic solutions	•	·	·	:	164
	3.2. The Hilbert spaces $H_t$		·	•		165
		•	•		•	165
	3.3. Structure of the spaces $H_t$	•	•	·		167
	3.5. Differentiability theorem	•	•	•		
	3.6. Solution of the equation $\mathbf{L}u = f$ .	•	٠	•		174
	Appendix I. The Projection Theorem $\dots$		•			175
	Appendix I. The Fredholm-Riesz-Schauder Theory	•	•		·	
	Appendix 11. The Freuhoim-Kiesz-Schauder Theory	•	•	•		183

xiv

## CONTENTS

4. Hilbert Space Approach. Dirichlet Problem	190
4.1. Introduction	190
4.2. Interior regularity $\cdot$	190
4.3. The spaces $H^t$ and $H_0^t$	192
4.4. Some lemmas in $H_0^t$	193
4.5. The generalized Dirichlet problem	196
4.6. Existence of weak solutions	198
4.7. Regularity at the boundary	200
4.8. Inequalities in a half-cube	202
Appendix. Analyticity of Solutions	207
5. Potential Theoretical Approach	211
5.1. Fundamental solutions. Parametrix	211
5.2. Some function spaces	216
5.3. Fundamental inequalities	220
5.4. Local existence theorem	228
5.5. Interior Schauder type estimates	231
5.6. Estimates up to the boundary	235
5.7. Applications to the Dirichlet problem	237
5.8. Smoothness of strong solutions	240
Appendix I. Proofs of the Fundamental Inequalities	242
Appendix II. Proofs of the Interpolation Lemmas	250
6. Function Theoretical Approach	254
6.1. Complex notation	255
6.2. Beltrami equation	257
6.3. A representation theorem	259
6.4. Consequences of the representation theorem	261
6.5. Two boundary value problems	263
Appendix. Properties of the Beltrami Equation. Privaloff's Theorem	267
7. Quasi-Linear Equations	282
7.1. Boundary value problems	282
7.2. Methods of solution	284
7.3. Examples	286
Bibliography	291
Supplement I. Eigenvalue Expansions, by Lars Gårding	301
Supplement II. Parabolic Equations, by A. N. MILGRAM	327
Index	341