

Contents

<i>Preface</i>	xiii
<i>Conventions and Notation</i>	xvii
1. Introduction to Wavelets	1
1.0 Introduction	1
1.1 The Essence of a Wavelet	2
Comments and Extensions to Section 1.1	4
1.2 The Essence of Wavelet Analysis	5
Comments and Extensions to Section 1.2	12
1.3 Beyond the CWT: the Discrete Wavelet Transform	12
Comments and Extensions to Section 1.3	19
2. Review of Fourier Theory and Filters	20
2.0 Introduction	20
2.1 Complex Variables and Complex Exponentials	20
2.2 Fourier Transform of Infinite Sequences	21
2.3 Convolution/Filtering of Infinite Sequences	24
2.4 Fourier Transform of Finite Sequences	28
2.5 Circular Convolution/Filtering of Finite Sequences	29
2.6 Periodized Filters	32
Comments and Extensions to Section 2.6	35
2.7 Summary of Fourier Theory	35
2.8 Exercises	39

3. Orthonormal Transforms of Time Series	41
3.0 Introduction	41
3.1 Basic Theory for Orthonormal Transforms	41
3.2 The Projection Theorem	44
3.3 Complex-Valued Transforms	45
3.4 The Orthonormal Discrete Fourier Transform	46
Comments and Extensions to Section 3.4	52
3.5 Summary	53
3.6 Exercises	54
4. The Discrete Wavelet Transform	56
4.0 Introduction	56
4.1 Qualitative Description of the DWT	57
Key Facts and Definitions in Section 4.1	67
Comments and Extensions to Section 4.1	68
4.2 The Wavelet Filter	68
Key Facts and Definitions in Section 4.2	74
Comments and Extensions to Section 4.2	75
4.3 The Scaling Filter	75
Key Facts and Definitions in Section 4.3	78
Comments and Extensions to Section 4.3	79
4.4 First Stage of the Pyramid Algorithm	80
Key Facts and Definitions in Section 4.4	86
Comments and Extensions to Section 4.4	87
4.5 Second Stage of the Pyramid Algorithm	88
Key Facts and Definitions in Section 4.5	93
4.6 General Stage of the Pyramid Algorithm	93
Key Facts and Definitions in Section 4.6	99
Comments and Extensions to Section 4.6	100
4.7 The Partial Discrete Wavelet Transform	104
4.8 Daubechies Wavelet and Scaling Filters: Form and Phase	105
Key Facts and Definitions in Section 4.8	116
Comments and Extensions to Section 4.8	117
4.9 Coiflet Wavelet and Scaling Filters: Form and Phase	123
4.10 Example: Electrocardiogram Data	125
Comments and Extensions to Section 4.10	134
4.11 Practical Considerations	135
Comments and Extensions to Section 4.11	145
4.12 Summary	150
4.13 Exercises	156

5. The Maximal Overlap Discrete Wavelet Transform	159
5.0 Introduction	159
5.1 Effect of Circular Shifts on the DWT	160
5.2 MODWT Wavelet and Scaling Filters	163
5.3 Basic Concepts for MODWT	164
Key Facts and Definitions in Section 5.3	168
5.4 Definition of j th Level MODWT Coefficients	169
Key Facts and Definitions in Section 5.4	173
Comments and Extensions to Section 5.4	174
5.5 Pyramid Algorithm for the MODWT	174
Key Facts and Definitions in Section 5.5	177
Comments and Extensions to Section 5.5	177
5.6 MODWT Analysis of ‘Bump’ Time Series	179
5.7 Example: Electrocardiogram Data	182
5.8 Example: Subtidal Sea Level Fluctuations	185
5.9 Example: Nile River Minima	190
5.10 Example: Ocean Shear Measurements	193
5.11 Practical Considerations	195
5.12 Summary	200
5.13 Exercises	204
6. The Discrete Wavelet Packet Transform	206
6.0 Introduction	206
6.1 Basic Concepts	207
Comments and Extensions to Section 6.1	217
6.2 Example: DWPT of Solar Physics Data	218
6.3 The Best Basis Algorithm	221
Comments and Extensions to Section 6.3	226
6.4 Example: Best Basis for Solar Physics Data	226
6.5 Time Shifts for Wavelet Packet Filters	229
Comments and Extensions to Section 6.5	231
6.6 Maximal Overlap Discrete Wavelet Packet Transform	231
6.7 Example: MODWPT of Solar Physics Data	234
6.8 Matching Pursuit	239
6.9 Example: Subtidal Sea Levels	243
Comments and Extensions to Section 6.9	247
6.10 Summary	247
6.11 Exercises	253
7. Random Variables and Stochastic Processes	255
7.0 Introduction	255
7.1 Univariate Random Variables and PDFs	256
7.2 Random Vectors and PDFs	258
7.3 A Bayesian Perspective	264
7.4 Stationary Stochastic Processes	266
7.5 Spectral Density Estimation	269

Comments and Extensions to Section 7.5	278
7.6 Definition and Models for Long Memory Processes	279
Comments and Extensions to Section 7.6	285
7.7 Nonstationary $1/f$ -Type Processes	287
Comments and Extensions to Section 7.7	289
7.8 Simulation of Stationary Processes	290
Comments and Extensions to Section 7.8	292
7.9 Simulation of Stationary Autoregressive Processes	292
7.10 Exercises	293
8. The Wavelet Variance	295
8.0 Introduction	295
8.1 Definition and Rationale for the Wavelet Variance	295
Comments and Extensions to Section 8.1	301
8.2 Basic Properties of the Wavelet Variance	304
Comments and Extensions to Section 8.2	306
8.3 Estimation of the Wavelet Variance	306
Comments and Extensions to Section 8.3	308
8.4 Confidence Intervals for the Wavelet Variance	311
Comments and Extensions to Section 8.4	315
8.5 Spectral Estimation via the Wavelet Variance	315
Comments and Extensions to Section 8.5	317
8.6 Example: Atomic Clock Deviates	317
8.7 Example: Subtidal Sea Level Fluctuations	324
8.8 Example: Nile River Minima	326
8.9 Example: Ocean Shear Measurements	327
8.10 Summary	335
8.11 Exercises	337
9. Analysis and Synthesis of Long Memory Processes	340
9.0 Introduction	340
9.1 Discrete Wavelet Transform of a Long Memory Process	341
Comments and Extensions to Section 9.1	350
9.2 Simulation of a Long Memory Process	355
Comments and Extensions to Section 9.2	361
9.3 MLEs for Stationary FD Processes	361
Comments and Extensions to Section 9.3	366
9.4 MLEs for Stationary or Nonstationary FD Processes	368
Comments and Extensions to Section 9.4	373
9.5 Least Squares Estimation for FD Processes	374
Comments and Extensions to Section 9.5	378
9.6 Testing for Homogeneity of Variance	379
Comments and Extensions to Section 9.6	382
9.7 Example: Atomic Clock Deviates	383
9.8 Example: Nile River Minima	386
9.9 Summary	388

9.10 Exercises	391
10. Wavelet-Based Signal Estimation	393
10.0 Introduction	393
10.1 Signal Representation via Wavelets	394
10.2 Signal Estimation via Thresholding	398
10.3 Stochastic Signal Estimation via Scaling	407
10.4 Stochastic Signal Estimation via Shrinkage	408
Comments and Extensions to Section 10.4	415
10.5 IID Gaussian Wavelet Coefficients	417
Comments and Extensions to Section 10.5	429
10.6 Uncorrelated Non-Gaussian Wavelet Coefficients	432
Comments and Extensions to Section 10.6	439
10.7 Correlated Gaussian Wavelet Coefficients	440
Comments and Extensions to Section 10.7	449
10.8 Clustering and Persistence of Wavelet Coefficients	450
10.9 Summary	452
10.10 Exercises	455
11. Wavelet Analysis of Finite Energy Signals	457
11.0 Introduction	457
11.1 Translation and Dilation	457
11.2 Scaling Functions and Approximation Spaces	459
Comments and Extensions to Section 11.2	462
11.3 Approximation of Finite Energy Signals	462
Comments and Extensions to Section 11.3	464
11.4 Two-Scale Relationships for Scaling Functions	464
11.5 Scaling Functions and Scaling Filters	469
Comments and Extensions to Section 11.5	472
11.6 Wavelet Functions and Detail Spaces	472
11.7 Wavelet Functions and Wavelet Filters	476
11.8 Multiresolution Analysis of Finite Energy Signals	478
11.9 Vanishing Moments	483
Comments and Extensions to Section 11.9	486
11.10 Spectral Factorization and Filter Coefficients	487
Comments and Extensions to Section 11.10	494
11.11 Summary	494
11.12 Exercises	500
Appendix. Answers to Embedded Exercises	501
References	552
Author Index	565
Subject Index	569