Contents

Preface vii

Chapter 1 Introduction **1**
1.1 Linearity and the beginning of time series analysis 1
1.2 Irregular time series and determinism 3
1.3 The objective of nonlinear time series analysis 4
1.4 Outline of the organisation of the present study 5

Chapter 2 Dynamical systems, time series and attractors **9**
2.1 Overview 9
2.2 Dynamical systems and state spaces 9
2.3 Measurements and time series 10
2.4 Deterministic dynamical systems 12
2.4.1 Attractors 12
2.4.2 Linear systems 15
2.4.3 Invariant measures 15
2.4.4 Sensitive dependence on initial conditions 18
2.4.5 Maps and discretised flows 19
2.4.6 Some important maps 21
2.4.7 Some important flows 24
2.5 Stochastic dynamical systems 28
2.5.1 Pure noise time series 29
2.5.2 Noise in dynamical systems 30
2.5.3 Linear stochastic systems 31
2.6 Nonstationarity 33

2.7 Experimental and observational time series 35
2.7.1 Electroencephalograms 36

Chapter 3 Linear methods **39**
3.1 Overview 39
3.2 Linear autocorrelation 39
3.3 Fourier spectrum estimation 40
3.3.1 Discrete Fourier transform and power spectrum 40
3.3.2 Practical application of Fourier spectrum estimation . . 42
3.4 Linear prediction and linear filtering 46

Chapter 4 State Space Reconstruction: Theoretical foundations **49**
4.1 Overview 49
4.2 The reconstruction problem 49
4.3 Definition of an embedding 51
4.4 Measures of the distortion due to embedding 52
4.5 The embedding theorem of Whitney and its generalisation . . . 53
4.6 Time-delay embedding 55
4.7 The embedding theorem of Takens and its generalisation 57
4.8 Some historical remarks 59
4.9 Filtered time-delay embedding 59
4.9.1 Derivatives and Legendre coordinates 60
4.9.2 Principal components: definition and properties 64
4.9.3 Principal components: applications 67
4.10 Other reconstruction methods 69
4.11 Interspike intervals 70

Chapter 5 State space reconstruction: Practical application **73**
5.1 Overview 73
5.2 The effect of noise on state space reconstruction 73
5.3 The choice of the time delay 75
5.4 In search of optimal embedding parameters 78
5.4.1 The Fillfactor algorithm 79
5.4.2 Comparing different reconstructions by PCA 82
5.4.3 The Integral Local Deformation (ILD) algorithm 84
5.4.4 Other algorithms for the estimation of optimal embedding parameters 87

Chapter 6 Dimensions: Basic definitions **93**
6.1 Overview 93
6.2 Why estimate dimensions? 94
6.3 Topological dimension 95
6.4 Hausdorff dimension 95
6.5 Capacity dimension 97
6.6 Generalisation of the Hausdorff dimension 98
6.7 Generalisation of capacity dimension 100
6.8 Information dimension 102
6.9 Continuous definition of generalised dimensions 103
6.10 Pointwise dimension 103
6.11 Invariance of dimension under reconstruction 104
6.12 Invariance of dimension under filtering 106
6.13 Methods for the calculation of dimensions 107
6.13.1 Box-counting algorithm 107
6.13.2 Pairwise-distance algorithm 109

Chapter 7 Lyapunov exponents and entropies **113**
7.1 Overview 113
7.2 Lyapunov exponents 113
7.3 Estimation of Lyapunov exponents from time series 115
7.4 Kaplan-Yorke dimension 116
7.5 Generalised entropies 117
7.6 Correlation entropy for time-delay embeddings 119
7.7 Pesin's theorem and partial dimensions 120

Chapter 8 Numerical estimation of the correlation dimension **123**
8.1 Overview 123
8.2 Correlation dimension as a tail parameter 123
8.3 Estimation of the correlation integral 124
8.4 Efficient implementations 126
8.5 The choice of metric 127
8.6 Typical behaviour of $C(r)$ 128
8.7 Dynamical range of $C(r)$ 131
8.8 Dimension estimation in the case of unknown embedding dimension 133
8.9 Global least squares approach 134

8.10 Chord estimator 136
8.11 Local-slopes approach 137
8.11.1 Implementation of the local-slopes approach 138
8.11.2 Typical behaviour of the local-slopes approach 138
8.12 Maximum-likelihood estimators 141
8.12.1 The Takens estimator 141
8.12.2 Extensions to the Takens estimator 143
8.12.3 The binomial estimator 144
8.12.4 The algorithm of Judd 145
8.13 Intrinsic dimension and nearest-neighbour algorithms 147

Chapter 9 Sources of error and data set size requirements 149
9.1 Overview 149
9.2 Classification of errors 149
9.3 Edge effects and singularities 151
9.3.1 Hypercubes with uniform measure 151
9.3.2 Underestimation due to edge effect 152
9.3.3 Data set size requirements for avoiding edge effects . . . 153
9.3.4 Distributions with singularities 155
9.4 Lacunarity 156
9.5 Additive measurement noise 158
9.6 Finite-resolution error 159
9.7 Autocorrelation error 160
9.7.1 Periodic-sampling error 161
9.7.2 Circles 164
9.7.3 Trajectory bias and temporal autocorrelation 166
9.7.4 Space-time separation plots 169
9.7.5 Quasiperiodic signals 169
9.7.6 Topological structure of N_t-tori 172
9.7.7 Autocorrelations in N_t-tori 173
9.7.8 Noise with power-law spectrum 175
9.7.9 Unrepresentativity error 178
9.8 Statistical error 178
9.9 Other estimates of data set size requirements 180

Chapter 10 Monte Carlo analysis of dimension estimation 183
10.1 Overview 183
10.2 Calibration systems 184

10.2.1 Mackey-Glass system 184
10.2.2 Gaussian white noise 186
10.2.3 Filtered noise 188
10.3 N_s-spheres 188
10.3.1 Analytical estimation of statistical error 189
10.3.2 Minimum data set size for N_s-spheres 192
10.3.3 Monte Carlo analysis of statistical error 194
10.3.4 Limited number of reference points 197
10.3.5 Comparison between GPA and JA 198
10.3.6 Results for maximum metric 200
10.4 Multiple Lorenz systems: True state space 202
10.4.1 Monte Carlo analysis of statistical error 203
10.4.2 Comparison between GPA and JA 206
10.4.3 Results for maximum metric 208
10.5 Multiple Lorenz systems: Reconstructed state space 209
10.5.1 Exact derivative coordinates 210
10.5.2 Time-delay coordinates 212
10.5.3 Hybrid coordinates 218

Chapter 11 Surrogate data tests **221**
11.1 Overview 221
11.2 Null hypotheses for surrogate data testing 222
11.3 Creation of surrogate data sets 224
11.3.1 Typical-realisation surrogates 224
11.3.2 Constrained-realisation surrogates 226
11.3.3 Surrogates with non-gaussian distribution 230
11.4 Refinements of constrained-realisation surrogate data set creation procedures 232
11.4.1 Improved AAPR surrogates 232
11.4.2 The wraparound artifact 234
11.4.3 Noisy sine waves 235
11.4.4 Limited phase randomisation 238
11.4.5 Remedies against the wraparound artifact 240
11.5 Evaluating the results of surrogate data tests 242
11.6 Interpretation of the results of surrogate data tests 244
11.7 Choice of the test statistic for surrogate data tests 245
11.8 Application of surrogate data testing to correlation dimension estimation 246

Chapter 12 Dimension analysis of the human EEG 249
12.1 Overview 249
12.2 The beginning of dimension analysis of the EEG 250
12.3 Application of dimension analysis to cerebral diseases and psychiatric disorders 251
12.3.1 EEG recordings from epileptic patients 252
12.3.2 EEG recordings from human sleep 252
12.4 Scepticism against finite dimension estimates from EEG recordings 254
12.4.1 Application of GPA to an EEG time series from sleep stage IV 255
12.4.2 Interpretation of the finite estimates found in the literature 258
12.5 Dimension analysis using moving windows 261
12.5.1 Application to nonstationary time series 262
12.5.2 Application to stationary time series 265
12.5.3 Application to a nonstationary EEG time series 267
12.6 Dimension analysis of EEG time series: Valuable or impractical? 271

Chapter 13 Testing for determinism in time series 273
13.1 Overview 273
13.2 The BDS-statistic 274
13.3 The dependence parameters δ_m by Savit & Green 277
13.3.1 Generalisations of the δ_m 280
13.3.2 Predictability parameters and the relationship between the δ_m and entropies 281
13.4 Testing for determinism and minimum embedding dimension . 283
13.5 Continuous versus discrete data sets 286
13.6 Reduction of EEG time series to discrete phase information . . 287
13.7 Savit-Green analysis of ISI series from multiple Lorenz systems 291
13.7.1 Distribution of the dependence parameters $\delta_m(r)$ 291
13.7.2 Surrogate data testing applied to the predictability parameters $\tilde{S}_m(r)$ 293
13.8 Savit-Green analysis of ISI series from nonstationary time series 296
13.9 Savit-Green analysis of ISI series from EEG time series 298
13.9.1 Analysis of an EEG time series from sleep stage IV . . 299
13.9.2 Analysis of a nonstationary EEG time series 301
13.10 Surrogate data testing of differenced time series 304

Chapter 14 Conclusion **309**

Table of notation 315

Bibliography 321

Index 337