

Contents

1. Introduction	1
a. Background	1
b. Variational definition of configurational forces	2
c. Interfacial energy. A further argument for a configurational force balance	5
d. Configurational forces as basic objects	7
e. The nature of configurational forces	9
f. Configurational stress and residual stress.	
Internal configurational forces	10
g. Configurational forces and indeterminacy	11
h. Scope of the book	12
i. On operational definitions and mathematics	12
j. General notation. Tensor analysis	13
j1. On direct notation	13
j2. Vectors and tensors. Fields	13
j3. Third-order tensors (3-tensors). The operation $T : \Lambda$.	15
j4. Functions of tensors	16
A. Configurational forces within a classical context	19
2. Kinematics	21
a. Reference body. Material points. Motions	21
b. Material and spatial vectors. The sets $\mathcal{E}_{\text{space}}$ and $\mathcal{E}_{\text{matter}}$	22
c. Material and spatial observers	23
d. Consistency requirement. Objective fields	23

3. Standard forces. Working	25
a. Forces	25
b. Working. Standard force and moment balances as consequences of invariance under changes in spatial observer	26
4. Migrating control volumes. Stationary and time-dependent changes in reference configuration	29
a. Migrating control volumes $P = P(t)$. Velocity fields for $\partial P(t)$ and $\partial \bar{P}(t)$	29
b. Change in reference configuration	31
b1. Stationary change in reference configuration	31
b2. Time-dependent change in reference configuration	32
5. Configurational forces	34
a. Configurational forces	34
b. Working revisited	35
c. Configurational force balance as a consequence of invariance under changes in material observer	36
d. Invariance under changes in velocity field for $\partial P(t)$. Configurational stress relation	37
e. Invariance under time-dependent changes in reference. External and internal force relations	38
f. Standard and configurational forms of the working. Power balance	39
6. Thermodynamics. Relation between bulk tension and energy.	
Eshelby identity	41
a. Mechanical version of the second law	41
b. Eshelby relation as a consequence of the second law	42
c. Thermomechanical theory	44
d. Fluids. Current configuration as reference	45
7. Inertia and kinetic energy. Alternative versions of the second law	46
a. Inertia and kinetic energy	46
b. Alternative forms of the second law	47
c. Pseudomomentum	47
d. Lyapunov relations	48
8. Change in reference configuration	50
a. Transformation laws for free energy and standard force	50
b. Transformation laws for configurational force	51
9. Elastic and thermoelastic materials	53
a. Mechanical theory	54
a1. Basic equations	54

a2.	Constitutive theory	54
b.	Thermomechanical theory	56
b1.	Basic equations	56
b2.	Constitutive theory	57
B.	The use of configurational forces to characterize coherent phase interfaces	61
10.	Interface kinematics	63
11.	Interface forces. Second law	66
a.	Interface forces	66
b.	Working	67
c.	Standard and configurational force balances at the interface . .	68
d.	Invariance under changes in velocity field for $\mathcal{S}(t)$. Normal configurational balance	69
e.	Power balance. Internal working	70
f.	Second law. Internal dissipation inequality for the interface . .	71
g.	Localizations using a pillbox argument	72
12.	Inertia. Basic equations for the interface	74
a.	Relative kinetic energy	74
b.	Determination of $b^{\mathcal{S}}$ and $e^{\mathcal{S}}$	75
c.	Standard and configurational balances with inertia	77
d.	Constitutive equation for the interface	78
e.	Summary of basic equations	79
f.	Global energy inequality. Lyapunov relations	80
C.	An equivalent formulation of the theory. Infinitesimal deformations	81
13.	Formulation within a classical context	83
a.	Background. Reason for an alternative formulation in terms of displacements	83
b.	Finite deformations. Modified Eshelby relation	84
c.	Infinitesimal deformations	86
14.	Coherent phase interfaces	88
a.	General theory	88
b.	Infinitesimal theory with linear stress-strain relations in bulk . .	89

D. Evolving interfaces neglecting bulk behavior	91
15. Evolving surfaces	93
a. Surfaces	93
a1. Background. Superficial stress	93
a2. Superficial tensor fields	94
b. Smoothly evolving surfaces	97
b1. Time derivative following \mathcal{S} . Normal time derivative..	97
b2. Velocity fields for the boundary curve $\partial\mathcal{G}$ of a smoothly	
evolving subsurface of \mathcal{S} . Transport theorem	99
b3. Transformation laws	100
16. Configurational force system. Working	101
a. Configurational forces. Working	101
b. Configurational force balance as a consequence of invariance	
under changes in material observer	102
c. Invariance under changes in velocity fields. Surface tension.	
Surface shear	103
d. Normal force balance. Intrinsic form for the working	104
e. Power balance. Internal working	105
17. Second law	108
18. Constitutive equations	110
a. Functions of orientation	110
b. Constitutive equations	111
c. Evolution equation for the interface	113
d. Lyapunov relations	114
19. Two-dimensional theory	115
a. Kinematics	115
b. Configurational forces. Working. Second law	116
c. Constitutive theory	118
d. Evolution equation for the interface	119
e. Corners	120
f. Angle-convexity. The Frank diagram	120
g. Convexity of the interfacial energy and evolution	
of the interface	124
E. Coherent phase interfaces with interfacial energy	
and deformation	127
20. Theory neglecting standard interfacial stress	129
a. Standard and configurational forces. Working	129

b.	Power balance. Internal working	131
c.	Second law	132
c1.	Second law. Interfacial dissipation inequality	132
c2.	Derivation of the interfacial dissipation inequality using a pillbox argument	132
d.	Constitutive equations	133
e.	Construction of the process used in restricting the constitutive equations	135
f.	Basic equations with inertial external forces	135
f1.	Standard and configurational balances	135
f2.	Summary of basic equations	136
g.	Global energy inequality. Lyapunov relations	137
21. General theory with standard and configurational stress within the interface		138
a.	Kinematics. Tangential deformation gradient	138
b.	Standard and configurational forces. Working	139
c.	Power balance. Internal working	142
d.	Second law. Interfacial dissipation inequality	144
e.	Constitutive equations	145
f.	Basic equations with inertial external forces	147
g.	Lyapunov relations	147
22. Two-dimensional theory with standard and configurational stress within the interface		149
a.	Kinematics	149
b.	Forces. Working	150
c.	Power balance. Internal working. Second law	152
d.	Constitutive equations	155
e.	Evolution equations for the interface	156
F. Solidification		157
23. Solidification. The Stefan condition as a consequence of the configurational force balance		159
a.	Single-phase theory	159
b.	The classical two-phase theory revisited. The Stefan condition as a consequence of the configurational balance	160
24. Solidification with interfacial energy and entropy		163
a.	General theory	163
b.	Approximate theory. The Gibbs-Thomson condition as a consequence of the configurational balance	166
c.	Free-boundary problems for the approximate theory. Growth theorems	167

c1. The quasilinear and quasistatic problems	167
c2. Growth theorems	168
G. Fracture	173
25. Cracked bodies	175
a. Smooth cracks. Control volumes	175
b. Derivatives following the tip. Tip integrals. Transport theorems .	177
26. Motions	182
27. Forces. Working	184
a. Forces	184
b. Working	186
c. Standard and configurational force balances	186
d. Inertial forces. Kinetic energy	188
28. The second law	190
a. Statement of the second law	190
b. The second law applied to crack control volumes	191
c. The second law applied to tip control volumes. Standard form of the second law	191
d. Tip traction. Energy release rate. Driving force	193
e. The standard momentum condition	194
29. Basic results for the crack tip	196
30. Constitutive theory for growing cracks	198
a. Constitutive relations at the tip	198
b. The Griffith-Irwin function	199
c. Constitutively isotropic crack tips. Tips with constant mobility .	200
31. Kinking and curving of cracks. Maximum dissipation criterion	201
a. Criterion for crack initiation. Kink angle	202
b. Maximum dissipation criterion for crack propagation	204
32. Fracture in three space dimensions (results)	208
H. Two-dimensional theory of corners and junctions neglecting inertia	211
33. Preliminaries. Transport theorems	213
a. Terminology	213
b. Transport theorems	214

b1.	Bulk fields	214
b2.	Interfacial fields	215
34. Thermomechanical theory of junctions and corners		218
a.	Motions	218
b.	Notation	219
c.	Forces. Working	220
d.	Second law	221
e.	Basic results for the junction	222
f.	Weak singularity conditions. Nonexistence of corners	222
g.	Constitutive equations	223
h.	Final junction conditions	224
I. Appendices on the principle of virtual work for coherent phase interfaces		225
A1. Weak principle of virtual work		227
a.	Virtual kinematics	227
b.	Forces. Weak principle of virtual work	228
c.	Proof of the weak theorem of virtual work	229
A2. Strong principle of virtual work		232
a.	Virtually migrating control volumes	232
b.	Forces. Strong principle of virtual work	233
c.	Proof of the strong theorem of virtual work	234
d.	Comparison of the strong and weak principles	236
References		239
Index		247