Contents

Preface v

1 Introduction 1

2 Instabilities and Patterns in Hydrodynamical Systems 15
- 2.1 Rayleigh-Bénard instability 15
- 2.2 Taylor-Couette instability 17
- 2.3 Liquid crystal instabilities 20

3 Instabilities and Patterns in Reaction-Diffusion Systems 25
- 3.1 Chemical instabilities 25
- 3.2 Defect microstructures in irradiated materials 31
- 3.3 Plastic deformation and dislocation patterns 32

4 Generic Aspects of Pattern-Forming Instabilities 37
- 4.1 Phenomenology 37
- 4.2 Reaction-diffusion dynamics and stability 39
- 4.3 Reduced dynamics and amplitude equations 41
- 4.4 Spatial patterns: selection and stability 43
  - 4.4.1 Isotropic systems 43
  - 4.4.2 Anisotropic systems 49
- 4.5 Phase dynamics of periodic patterns 52
  - 4.5.1 Isotropic systems 52
  - 4.5.2 Anisotropic systems 59

5 The Hopf Bifurcation and Related Spatio-Temporal Patterns **65**
5.1 The generic aspects of oscillatory media 65
5.1.1 The complex Ginzburg-Landau equation 65
5.1.2 Phase dynamics and spiral waves 67
5.2 Real chemical systems and the complex Ginzburg-Landau equation 72
5.2.1 Determination of the CGLE parameters in real systems . 72
5.2.2 CGLE parameters of the BZ reaction 75
5.3 The effect of natural forcings on chemical oscillators 77
5.3.1 The effect of convection on chemical waves 77
5.3.2 The effect of vertical gradients on chemical oscillations . 79
5.4 Conclusions 82

6 The Turing Instability and Associated Spatial Structures **87**
6.1 The Turing mechanism 87
6.2 The search for Turing structures 90
6.2.1 Convectively driven chemical patterns 90
6.2.2 Double diffusion and chemical fingers 91
6.3 At last, genuine Turing structures? 93
6.4 The interaction between Turing and Hopf instabilities 98
6.4.1 Amplitude equations for Turing-Hopf modes 98
6.4.2 Pattern selection for codimension-2 Turing-Hopf bifurcations 99
6.4.3 Defects, defect bifurcations and localized structures . . . 102

7 Defects and Defect Bifurcations **107**
7.1 Generic existence of defects 108
7.2 Examples of defects 111
7.2.1 Codimension-1 defects 111
7.2.2 Codimension-2 defects 112
7.3 Defects and disorder 114
7.4 Bifurcation of defects 121

8 The Effect of External Fields **127**
8.1 Spatial forcing of stationary patterns 129
8.1.1 Resonant forcings 129
8.1.2 Near-resonant forcings and commensurate incommensurate transitions 130
8.2 Temporal forcing of a Hopf bifurcation 133
8.3 Temporal forcing of one-dimensional wave patterns 135
8.3.1 Pattern selection and defects 135
8.3.2 Experimental observations 146
8.4 Temporal forcing of two-dimensional wave patterns 147
8.4.1 Isotropic systems 147
8.4.2 Anisotropic systems 148

8.5 Spatial forcing of wave patterns 151
8.6 Flow field effects on pattern forming instabilities 155
8.7 The effect of noise on wave patterns 158
8.8 Conclusions 162

9 Fronts **169**
9.1 One-dimensional aspects 171
9.1.1 The leading-edge approach 171
9.1.2 Breakdown of the leading-edge approach 174
9.1.3 Envelope fronts 174
9.1.4 Multiple fronts 177
9.2 Two-dimensional aspects 179
9.2.1 Propagation of roll patterns 181
9.2.2 Propagation of hexagonal patterns 184

10 Pattern Formation: Generic versus Nongeneric Aspects **189**
10.1 Kinetic coefficients 189
10.2 Nongradient dynamics 190
10.3 Experimental set-ups 192

11 Microstructures in Irradiated Materials **197**
11.1 Particle irradiation of metals and alloys 197
11.1.1 A rate theory model for microstructure evolution under irradiation 198
11.1.2 Dislocation loop dynamics 200
11.1.3 The linear stability analysis 204
11.1.4 The weakly nonlinear regime 205
11.1.5 Numerical analysis 209
11.1.6 Conclusions 216
11.2 Laser induced deformation of surfaces 217
11.2.1 Thin film dynamics under laser irradiation 221
11.2.2 Linear stability analysis. 222
11.2.3 Weakly nonlinear analysis 224
11.2.4 Finite size effects. 227
11.2.5 Conclusions 228

12 Plastic Instabilities **231**
12.1 Dislocation dynamics and rate equations 232
12.2 Stability analysis and bifurcations 234
12.3 Nonlinear analysis 236
12.4 Multiple slip 237

13 Afterword **241**

14 Appendices **245**
14.1 Bifurcations and normal forms 245

14.1.1 Bifurcations 245
14.1.2 Stability of equilibria 248
14.1.3 Lyapounov functions 250
14.1.4 Typical bifurcation examples 252
14.1.5 The Hopf bifurcation theorem 255
14.1.6 The Center Manifold Theorem 257
14.2 More about dynamical models 258
14.2.1 Proctor-Sivashinsky 258
14.2.2 Ginzburg-Landau 269
14.3 The Brusselator: a toy model for pattern formation in RD systems 275
14.3.1 The Turing instability 277
14.3.2 The Hopf Instability 289
14.3.3 Amplitude equations for Turing-Hopf modes 292
14.4 Resonant forcings of nonlinear oscillators 293
14.4.1 Parametric forcing 293
14.4.2 Strong resonances in spatially extended systems 295
14.4.3 Stationary solutions and resonance horns 296
14.4.4 From oscillations to excitability 298

Index **303**