
Contents

1	Introduction	1
1.1	Dynamical models	1
1.2	Celestial mechanics	5
1.3	Lorenz: the end of weather prediction?	7
1.4	Complex behaviour of simple systems	11
2	Orbits of one-dimensional systems	13
2.1	Discrete dynamical systems	13
2.2	Fixed points and stability	19
2.3	Some orbits of the tent map	24
2.4	Period doubling of the logistic map	30
2.5	Periodic orbits and compositions	34
2.6	The fully chaotic tent map	37
2.7	Numerical versus exact orbits	41
2.8	Fourier analysis of an orbit	44
2.9	Lyapunov exponent of an orbit	51
2.10	Chaotic orbits	55
2.11	Ergodic orbits	60
3	Bifurcations in one-dimensional systems	65
3.1	Bifurcation diagrams	65
3.2	Final state diagrams	67
3.3	Period doubling mechanism	71
3.4	Period doubling cascades	79
3.5	Feigenbaum's universal constants	82
3.6	Tangent bifurcations	89
3.7	Intermittent behaviour	92
3.8	Unstable orbits and crises	96
3.9	Transcritical and pitchfork bifurcations	100
3.10	Theory of Feigenbaum scaling	103
4	Two-dimensional systems	107
4.1	The Hénon map	107
4.2	Fixed points	113

4.3	Area contraction	116
4.4	Stability of fixed points	119
4.5	Lyapunov exponents	127
4.6	Basin boundaries	132
4.7	Return Maps	136
4.8	Linear feedback and control	140
4.9	Producing the strange attractor	145
5	Fractals	147
5.1	The Hénon attractor	148
5.2	The Cantor Set	150
5.3	Fractal bifurcation diagrams	155
5.4	Capacity dimension	157
5.5	Capacity dimension of the Hénon attractor	161
5.6	Self-similar fractals	165
5.7	Lyapunov dimension of two-dimensional maps	167
5.8	The Rössler attractor	170
5.9	The Feigenbaum attractor	172
6	Non-linear oscillations	177
6.1	The driven non-linear pendulum	177
6.2	Phase Plane	182
6.3	Poincaré sections	186
6.4	Lyapunov exponents	191
A	Chaos for Java Software	197
A.1	Installation	197
A.2	General features	198
A.3	BIFURCATION DIAGRAMS	205
A.4	FOURIER ANALYSIS	206
A.5	GRAPHICAL ANALYSIS	207
A.6	ITERATE(1D)	209
A.7	ITERATE(2D)	211
A.8	LYAPUNOV EXPONENTS	214
A.9	ODE ORBITS	215
A.10	POINCARÉ SECTIONS	217
A.11	RETURN MAPS	218
B	Discrete Fourier Transform	221
B.1	Complex roots of unity	221
B.2	Discrete orthogonality	222
B.3	Fourier amplitudes	222
B.4	Using real and imaginary parts	223

B.5	Fast Fourier Transform	224
C	Variational equations	225
C.1	Derivation and meaning	225
C.2	The area contracting property	225
C.3	Three-dimensional case	226
D	List of Maps and Differential Equations	227
D.1	One-dimensional maps	227
D.2	Two-dimensional maps	228
D.3	Differential equations	228
Bibliography		229
Index		232