

CONTENTS

CHAPTER 1 Dynamics of Particles and Fields

1.1	Beams and Beam Physics	1
1.2	Differential Equations, Determinism, and Maps	2
1.2.1	Existence and Uniqueness of Solutions	3
1.2.2	Maps of Deterministic Differential Equations	6
1.3	Lagrangian Systems	7
1.3.1	Existence and Uniqueness of Lagrangians	7
1.3.2	Canonical Transformation of Lagrangians	8
1.3.3	Connection to a Variational Principle	12
1.3.4	Lagrangians for Particular Systems	14
1.4	Hamiltonian Systems	20
1.4.1	Manipulations of the Independent Variable	23
1.4.2	Existence and Uniqueness of Hamiltonians	26
1.4.3	The Duality of Hamiltonians and Lagrangians	27
1.4.4	Hamiltonians for Particular Systems	32
1.4.5	Canonical Transformation of Hamiltonians	34
1.4.6	Universal Existence of Generating Functions	44
1.4.7	Flows of Hamiltonian Systems	53
1.4.8	Generating Functions	54
1.4.9	Time-Dependent Canonical Transformations	59
1.4.10	The Hamilton–Jacobi Equation	60
1.5	Fields and Potentials	62
1.5.1	Maxwell’s Equations	62
1.5.2	Scalar and Vector Potentials	65
1.5.3	Boundary Value Problems	76

CHAPTER 2 Differential Algebraic Techniques

2.1	Function Spaces and Their Algebras	81
2.1.1	Floating Point Numbers and Intervals	81
2.1.2	Representations of Functions	82

2.1.3	Algebras and Differential Algebras	84
2.2	Taylor Differential Algebras	85
2.2.1	The Minimal Differential Algebra	85
2.2.2	The Differential Algebra $n D_v$	91
2.2.3	Generators, Bases, and Order	92
2.2.4	The Tower of Ideals, Nilpotency, and Fixed Points	96
2.3	Advanced Methods	100
2.3.1	Composition and Inversion	100
2.3.2	Important Elementary Functions	102
2.3.3	Power Series on $n D_v$	104
2.3.4	ODE and PDE Solvers	108
2.3.5	The Levi-Civita Field	111

CHAPTER 3 Fields

3.1	Analytic Field Representation	120
3.1.1	Fields with Straight Reference Orbit	120
3.1.2	Fields with Planar Reference Orbit	125
3.2	Practical Utilization of Field Information	126
3.2.1	Multipole Measurements	127
3.2.2	Midplane Field Measurements	128
3.2.3	Electric Image Charge Methods	129
3.2.4	Magnetic Image Charge Methods	132
3.2.5	The Method of Wire Currents	139

CHAPTER 4 Maps: Properties

4.1	Manipulations	146
4.1.1	Composition and Inversion	146
4.1.2	Reversion	147
4.2	Symmetries	148
4.2.1	Midplane Symmetry	148
4.2.2	Rotational Symmetry	150
4.2.3	Symplectic Symmetry	155
4.3	Representations	159
4.3.1	Flow Factorizations	159
4.3.2	Generating Functions	164

CHAPTER 5 Maps: Calculation

5.1	The Particle Optical Equations of Motion	168
5.1.1	Curvilinear Coordinates	168

5.1.2	The Lagrangian and Lagrange's Equations in Curvilinear Coordinates	174
5.1.3	The Hamiltonian and Hamilton's Equations in Curvilinear Coordinates	179
5.1.4	Arc Length as an Independent Variable for the Hamiltonian	185
5.1.5	Curvilinear Coordinates for Planar Motion	188
5.2	Equations of Motion for Spin	190
5.3	Maps Determined by Algebraic Relations	195
5.3.1	Lens Optics	195
5.3.2	The Dipole	197
5.3.3	Drifts and Kicks	201
5.4	Maps Determined by Differential Equations	201
5.4.1	Differentiating ODE Solvers	201
5.4.2	DA Solvers for Differential Equations	202
5.4.3	Fast Perturbative Approximations	203

CHAPTER 6

Imaging Systems

6.1	Introduction	211
6.2	Aberrations and their Correction	214
6.3	Reconstructive Correction of Aberrations	217
6.3.1	Trajectory Reconstruction	218
6.3.2	Reconstruction in Energy Loss Mode	221
6.3.3	Examples and Applications	223
6.4	Aberration Correction via Repetitive Symmetry	228
6.4.1	Second-Order Achromats	230
6.4.2	Map Representations	233
6.4.3	Major Correction Theorems	238
6.4.4	A Snake-Shaped Third-Order Achromat	240
6.4.5	Repetitive Third- and Fifth Order Achromats	243

CHAPTER 7

Repetitive Systems

7.1	Linear Theory	250
7.1.1	The Stability of the Linear Motion	250
7.1.2	The Invariant Ellipse of Stable Symplectic Motion	260
7.1.3	Transformations of Elliptical Phase Space	262
7.2	Parameter-Dependent Linear Theory	265
7.2.1	The Closed Orbit	266
7.2.2	Parameter Tune Shifts	267
7.2.3	Chromaticity Correction	268
7.3	Normal Forms	270
7.3.1	The DA Normal Form Algorithm	270
7.3.2	Symplectic Systems	274
7.3.3	Nonsymplectic Systems	277
7.3.4	Amplitude Tune Shifts and Resonances	279

7.3.5	Invariants and Stability Estimates	282
7.3.6	Spin Normal Forms	288
7.4	Symplectic Tracking	292
7.4.1	Generating Functions	293
7.4.2	Prefactorization and Symplectic Extension	295
7.4.3	Superposition of Local Generators	297
7.4.4	Factorizations in Integrable Symplectic Maps	298
7.4.5	Spin Tracking	299