



# Contents

<b>1. Introduction . . . . .</b>	<b>1</b>
1.1 Goal . . . . .	1
1.2 Physiological Motivation . . . . .	3
1.2.1 Resetting Cerebral Rhythms . . . . .	3
1.2.2 Deep Brain Stimulation . . . . .	5
1.3 Stochastic Approach . . . . .	6
1.4 Synergetics . . . . .	9
<b>2. Resetting an Ensemble of Oscillators . . . . .</b>	<b>11</b>
2.1 Introductory Remarks . . . . .	11
2.2 Deterministic Models . . . . .	11
2.2.1 Macroscopic Level . . . . .	12
2.2.2 Cluster of Oscillators . . . . .	19
2.3 Stochastic Model . . . . .	21
2.4 Fokker–Planck Equation . . . . .	22
2.4.1 Stationary Solution . . . . .	23
2.4.2 Fourier Transformation . . . . .	24
2.5 Spontaneous Behavior . . . . .	25
2.6 Black Holes Without Noise . . . . .	26
2.7 Ensemble Dynamics During Stimulation . . . . .	28
2.7.1 Black Holes in the Presence of Noise . . . . .	28
2.7.2 Stimulation Induced Frequency Shift . . . . .	37
2.7.3 Stimulation Mechanism with Higher Harmonics . . . . .	39
2.8 Firing Patterns . . . . .	46
2.9 Summary and Discussion . . . . .	51
<b>3. Synchronization Patterns . . . . .</b>	<b>55</b>
3.1 Introductory Remarks . . . . .	55
3.2 Pattern Recognition . . . . .	55
3.3 Clustering . . . . .	57
3.4 Populations of Neurons . . . . .	58
3.4.1 Model Neuron . . . . .	59
3.4.2 Neuronal Interactions . . . . .	60
3.5 Populations of Phase Oscillators . . . . .	61

3.6	Slaving Principle and Center Manifold . . . . .	64
3.6.1	Center Manifold Theorem . . . . .	66
3.6.2	Strategy . . . . .	69
3.7	$n$ -Cluster States . . . . .	69
3.7.1	Configuration of Cluster States . . . . .	69
3.7.2	One Cluster . . . . .	71
3.7.3	Two Clusters . . . . .	75
3.7.4	Three Clusters . . . . .	77
3.7.5	Four Clusters . . . . .	78
3.8	Complexity of Synchronized States . . . . .	79
3.8.1	Hierarchy of Frequency Levels . . . . .	79
3.8.2	Phase and Frequency Shifts . . . . .	82
3.8.3	Cluster Variables . . . . .	84
3.8.4	Frozen States . . . . .	86
3.8.5	Transient Behavior . . . . .	88
3.8.6	Coupling Mechanism . . . . .	90
3.9	Neural Coding . . . . .	93
3.9.1	Information Compression . . . . .	93
3.9.2	Coding by Clusters . . . . .	93
3.10	Summary . . . . .	96
<b>4.</b>	<b>Stochastic Model . . . . .</b>	<b>99</b>
4.1	Introductory Remarks . . . . .	99
4.2	Derivation of the Model Equation . . . . .	100
4.3	Fourier Transformation . . . . .	105
4.4	Summary and Discussion . . . . .	106
<b>5.</b>	<b>Clustering in the Presence of Noise . . . . .</b>	<b>107</b>
5.1	Introductory Remarks . . . . .	107
5.2	Modelling Emerging Synchronization . . . . .	107
5.3	Comparison with the Ensemble's Dynamics . . . . .	108
5.4	Noisy Cluster States . . . . .	109
5.4.1	Linear Problem . . . . .	110
5.4.2	First-Mode Instability . . . . .	111
5.4.3	Second-Mode Instability . . . . .	117
5.4.4	Third-Mode Instability . . . . .	118
5.4.5	Fourth-Mode Instability . . . . .	119
5.4.6	Two-Mode Instability . . . . .	120
5.5	Scaling of Noisy Cluster States . . . . .	127
5.6	The Experimentalist's Inverse Problem . . . . .	130
5.6.1	Travelling Waves . . . . .	130
5.6.2	Firing Patterns . . . . .	132
5.7	Neural Coding Revisited . . . . .	135
5.8	Summary and Discussion . . . . .	137

<b>6. Single Pulse Stimulation . . . . .</b>	145
6.1 Introductory Remarks . . . . .	145
6.2 How Stimulation Affects Order Parameters . . . . .	147
6.2.1 Cluster Variables and Order Parameters . . . . .	148
6.2.2 Uniform and Partial Desynchronization . . . . .	149
6.2.3 Stimulating a One-Cluster State . . . . .	150
6.2.4 Stimulating a Two-Cluster State . . . . .	159
6.3 Transient Mode Excitation and Early Response . . . . .	163
6.3.1 Excitation of Higher Order . . . . .	165
6.3.2 Excitation of Lower Order . . . . .	169
6.4 Couplings Determine Reaction to Stimulation . . . . .	172
6.4.1 Rapid Recovery . . . . .	173
6.4.2 Harmonic Early Response . . . . .	174
6.5 Vulnerability and Recovery . . . . .	176
6.5.1 Phase Errors Versus Duration Errors . . . . .	177
6.5.2 Protective Effect of Couplings . . . . .	181
6.5.3 Partial Desynchronization and Transient Phenomena . . . . .	183
6.6 Black Hole and Recovery . . . . .	185
6.7 Subcritical Long Pulses . . . . .	187
6.7.1 Spiraling Towards the Desynchronized State . . . . .	189
6.7.2 Excitation of Higher Order . . . . .	191
6.7.3 Excitation of Lower Order . . . . .	195
6.8 Summary and Discussion . . . . .	195
<b>7. Periodic Stimulation . . . . .</b>	203
7.1 Introductory Remarks . . . . .	203
7.2 Smooth Periodic Stimulation . . . . .	204
7.2.1 1:1 Phase Locking . . . . .	205
7.2.2 1:2 Phase Locking . . . . .	207
7.2.3 Changes of the Synchronization Pattern . . . . .	210
7.3 Pulsatile Periodic Stimulation . . . . .	212
7.4 Annihilation of Rhythms . . . . .	214
7.5 Summary and Discussion . . . . .	215
<b>8. Data Analysis . . . . .</b>	219
8.1 Introductory Remarks . . . . .	219
8.2 Phases and Amplitudes . . . . .	220
8.2.1 Marker Events . . . . .	221
8.2.2 Reconstruction of the Modes' Dynamics . . . . .	222
8.2.3 Slaving Principle and Transients . . . . .	224
8.3 Tracking Down the Black Holes . . . . .	225
8.4 MEG and EEG Analysis . . . . .	226
8.4.1 Triggered Averaging . . . . .	227
8.4.2 Phase Dependent Triggered Averaging . . . . .	227
8.4.3 Stimulus Locked $n:m$ Transients . . . . .	229

8.4.4	Stimulus Locked $n : m$ Transients with Delay .....	236
8.4.5	Multiple Stimulus Locked $n : m$ Transients .....	238
8.4.6	Detection of Multiple Stimulus Locked $n : m$ Transients .....	240
8.4.7	Multiple Stimulus Locked $n : m$ Transients with Delay .....	242
8.4.8	$n : m$ Phase Synchronization .....	244
8.4.9	Self-Synchronization Versus Transients .....	248
8.4.10	The Flow of Synchronized Cerebral Activity .....	249
8.4.11	Inverse Problems .....	250
8.5	Summary and Discussion .....	251
<b>9.</b>	<b>Modelling Perspectives</b> .....	257
9.1	Neural Oscillators .....	257
9.1.1	Time-Delayed Interactions .....	258
9.1.2	Anatomy of Interacting Clusters .....	260
9.2	Limit Cycle Oscillators .....	260
9.3	Chaotic Oscillators .....	261
9.4	Macroscopic Versus Microscopic .....	262
<b>10.</b>	<b>Neurological Perspectives</b> .....	265
10.1	Therapeutic Stimulation Techniques .....	265
10.2	Parkinsonian Resting Tremor .....	266
10.2.1	Disease Mechanism .....	266
10.2.2	Stereotactic Treatment .....	270
10.2.3	Resetting the Tremor Rhythm .....	272
<b>11.</b>	<b>Epilogue</b> .....	275
11.1	Natural Sensory Stimulation .....	275
11.2	Experimental Electrical and Magnetic Stimulation .....	279
11.3	Therapeutic Stimulation .....	280
<b>Appendices</b>	.....	283
<b>A.</b>	<b>Numerical Analysis of the Partial Differential Equations</b> ..	285
<b>B.</b>	<b>Phase and Frequency Shifts Occurring in Chap. 3</b> ..	287
B.1	Two Clusters .....	287
B.2	Three Clusters .....	288
B.3	Four Clusters .....	288
<b>C.</b>	<b>Single-Mode Instability</b> .....	291
C.1	First-Mode Instability .....	291
C.2	Second-Mode Instability .....	291
C.3	Third-Mode Instability .....	292
C.4	Fourth-Mode Instability .....	292

<b>D. Two-Mode Instability .....</b>	293
D.1 Center Manifold .....	293
D.2 Order Parameter Equation .....	293
D.3 Linear Problem (Type I) .....	294
D.4 Linear Problem (Type II) .....	294
D.5 Singularities .....	295
<b>References .....</b>	297
<b>Author Index .....</b>	321
<b>Subject Index .....</b>	325